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Feeding-marketing decisions and the value of price forecast information to the cattle feeder

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I. INTRODUCTION

The profitability of a cattle feeding enterprise can be attributed to the decisions made and the resulting consequences. Many of the decisions made by the cattle feeder are made under conditions of uncertainty. Uncertainty occurs when the outcome of a decision is random; the decision maker is not at all certain what the outcome will be.

Bullock and Logan (9, p. 3) identify three sources of uncertainty in cattle feeding as: (1) cattle prices, both buying and selling, (2) feed prices, and (3) feedlot performance. The latter includes both daily rate of gain and anticipated slaughter grade. This thesis concerns feeding and marketing decisions as affected by cattle prices and feed prices. It is assumed that the cattle feeder has had adequate experience to accurately predict the performance of cattle being fed.

The wide variation in profits derived from cattle feeding enterprises in Iowa exemplifies the existing uncertainty. For choice steer calves fed to slaughter weights and marketed between January 1973 and December 1974, net returns ranged from $161.31 loss to $130.03 profit per head (10). For choice yearling steers marketed during the same period net returns varied from $161.12 loss to $118.89 profit per head.

Price forecast information is available from several sources to assist the cattle feeder in decision making. Subscription fees for price forecast information vary depending on the source. Market information provided by a governmental agency is typically available at no cost while information provided by some extension groups or private agencies is available by paying a subscription fee.

A. Objectives

1. Feeding and marketing decisions

Cattle feeders are continuously making and re-evaluating decisions involving both production and marketing aspects of the enterprise. The two aspects are not independent nor mutually exclusive. Production decisions involve: (1) type of feedlot facilities, (2) whether to feed or not feed cattle, (3) what ration to feed, (4) what rate of gain to achieve, (5) how long to feed cattle, and (6) what type of cattle to feed with reference to sex, weight, and quality. Marketing decisions involve: (1) purchasing decisions, (2) selling decisions, and (3) utilization of price forecast information.

Purchasing decisions involve timing of purchases to complement other enterprises in allocation of resources and to take advantage of seasonal or cyclical aspects of cattle markets. In addition to timing of purchases, the feeder must determine the source from which to purchase cattle -- order buyer, terminal, auction, grower, cow-calf operator -- or to produce the cattle himself.

Selling decisions, similar to purchase, are also concerned with time aspects. In addition, the feeder must determine the weight range at which cattle will be sold. The feeder must decide on method and condition of sale. Method concerns selling cattle direct to a packer, through a terminal market, or an auction outlet. Condition of sale refers to basis on which an animal is sold (live or carcass weight).

Another decision containing both production and marketing aspects of cattle feeding is utilization and incorporation of price forecast information into the decision process. First, it is necessary to determine if price forecast information will be used and from what source the information is to be obtained. Second, it is necessary to determine how the forecast information will be incorporated into the decision making model.

The first objective of this dissertation is to develop an economic decision model incorporating uncertainty to assist the cattle feeder in making both production and marketing decisions that maximize expected returns above variable costs. The model will provide answers to such questions as:

(1) Given feed prices and livestock price expectations, should cattle be fed or continued on feed; if so, for how long?

(2) What is the optimal rate of gain to achieve?

(3) What is the composition and cost of the least cost ration that provides the optimal rate of gain?

Variables that influence the profitability of a cattle feeding enterprise are: (1) price paid for feeder animals, (2) price received per pound for slaughter weight animals, (3) amount of weight put on the animal during the feeding period, (4) ration costs, (5) other variable costs such as labor, yardage fees, veterinary and medical, etc., and (6) fixed costs.

Given feed ingredient costs and cattle price expectations, the feeder must decide how long to feed cattle and the profit maximizing daily rate of gain. Rate of gain and length of time to feed cattle are not independent decisions.

2. Economic value of price forecast information

Knowledge of the economic value of price forecast information could be useful to both the user and the providing agency. For the user, two questions may arise concerning the availability and use of price forecast information. First, what is the expected income change from incorporating price forecast information into the decision model, and secondly, does the expected value or the realized value of the forecast information exceed the cost of the information.

The providing agency may be interested in determining the economic value of price forecast information for two reasons. First, to compare subscription fees with the expected value of information. An adjustment of fees may be appropriate if the expected value of the information differs from the subscription fee. Secondly, the economic value of price forecast information might be useful in justification of request for, or allocation of, funds to some market information sources.

The second objective of this dissertation is to determine the economic value of cattle price forecast information appearing in the Iowa Farm Outlook letter. The letter is published two times per month by the Iowa Cooperative Extension Service and is available for $2.00 per year.

B. Procedure Summary

The following procedures were used to accomplish the objectives listed in the preceding section:

For objective one:

A Bayesian decision model was used to determine the optimal strategies in feeding and marketing cattle. A linear programming model was developed to determine the least cost ration for each cattle feeding activity considered. Results of a linear programming model were used to compute payoffs in the Bayesian decision model.

For objective two:

Value of information appearing in the Iowa Farm Outlook letter for a prespecified period of time was determined using Bayesian decision theory techniques. A comparison of income above variable costs from following two types of Bayesian strategies and a more "naive" strategy was made. The latter strategy assumed outlook information contained no forecast error and allowed no variation in ration composition.

C. Following Chapters

Chapter II contains a discussion of Bayesian decision theory, illustrates a Bayesian decision model, and reviews some applied studies. Chapter III is a presentation of the general model used in this thesis and results of submodels used as input into the main model. Chapter IV presents the results of Bayesian decision strategies, the "naive" strategy, value of outlook information, and comparisons of income flows between strategies. Chapter V contains the summary and conclusions.

II. BAYESIAN DECISION MODEL

A decision problem arises when a person is confronted with an alternative or series of alternative courses of actions, one of which must be selected. The decision may be as simple as deciding whether or not to engage in a specific activity or do a specific thing where the outcome is known with certainty, or it may be more complicated, as where the decision involves selecting one or a combination of activities from several possible alternatives where the outcomes may not be known with certainty.

Numerous models have been developed to assist in decision making under uncertainty. Each model prescribes a precise criterion which for any decision problem, unambiguously selects the act(s) which is (are) tautologically termed "optimal according to the criterion" (27, p. 278). Basic components of all decision problems involving uncertainty are: (1) alternative actions from within a set, $(A_i \varepsilon A)$, available to the decision maker, (2) the set of states of the world, $(\theta_j \varepsilon \Theta)$, and (3) a consequence or payoff u_{ij} associated with each combination $A_i\theta_j$. The decision problem under uncertainty is illustrated in table 2.1. The decision problem reduces to: given an m by n array of numbers u_{ij} to choose a row (act) which is optimal in some sense -- or more generally, to rank the rows (acts) according to some optimality criterion (27, p. 276).

The model used in this dissertation to solve decision problems under uncertainty is a Bayesian decision model. The motivation for using the Bayesian model is essentially to incorporate any and all

Table 2.1. Basic components of a decision problem

	State of the world					
Acts	θ_1	θ_2	....	θ_j	....	θ_n
A_1	u_{11}	u_{12}	....	u_{1j}	....	u_{1n}
A_2	u_{21}	u_{22}	....	u_{2j}	....	u_{2n}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
A_i	u_{i1}	u_{i2}	....	u_{ij}	....	u_{in}
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
A_m	u_{m1}	u_{m2}		u_{mj}		u_{mn}

available information, whether it be sample information or information of some other nature, into the decision problem to assist the decision maker in selecting the action that maximizes the expected outcome -- whether expressed in utility or monetary terms -- when probabilities have been assigned or computed for states of the world (21, p. 445). The mechanism used to combine all available information is Bayes' theorem.

In this chapter the components of a Bayesian decision model are presented, the computation of a Bayesian strategy and determination of expected and conditional values and expected net gain of sample information are illustrated, a discussion on attitudes towards risk and selection of a decision criteria is presented, and a review of selected studies utilizing Bayesian decision models is presented. For additional readings and a more detailed description of Bayesian decision models, the reader is referred to Chernoff and Moses, Luce and Raiffa, Raiffa and Schlaifer, and Hays and Winkler (12, 27, 36, 21).

A. Components of a Bayesian Decision Model

The following are essential components of a Bayesian decision model and are required to determine the Bayesian strategy:

(1) Set of actions (A) available to the decision maker; the decision maker is confronted with selecting from the set A one act A_i ($i = 1, \ldots, m$) that in some way appears "best" to him ($A_i \epsilon A$).

(2) States of the world (Θ) confronting the decision maker. The state of the world θ_j ($j = 1, \ldots, n$) is a random event that influences the outcome of action A_i. The decision maker has no control or input in determining which θ_j will occur ($\theta_j \epsilon \Theta$).

(3) Consequences $[C(A_i\theta_j)]$ or gains $[G(A_i\theta_j)]$ associated with each combination of action and state of the world. The consequence of any action and state of the world are typically expressed in terms of gains (g_{ij}), losses or regrets (r_{ij}), or utility (u_{ij}). The latter term was used to describe the consequences of the decision problem appearing in table 2.1.

The three components listed above are similar to the components discussed previously in this chapter. These are all of the required components for game theoretic models but are only part of the required components of a Bayesian decision model. In the latter model it is assumed that the decision maker is not in "complete ignorance" about which state of the world will occur; he has some idea about the long-run frequencies with which the various states of the world will occur.

(4) Probability distribution $P(\theta)$ or $P(\theta|Z)$ over states of the world θ. The first probability is the "prior" probability distribution. The prior reflects all relevant information concerning va-ious states of the world θ before collecting or incorporating sample information into the decision. The prior probability can be either of an objective or subjective form. The objective form of probability is typically associated with the frequency interpretation of probability; it is based on results of previous samples or observations. Subjective probability is a measure of one's "degree of belief;" it is a personal quantified judgment of a particular individual.

The second type probability used in Bayesian decision models is the "posterior" probability, $P(\theta|Z)$. The posterior probability combines all relevant information currently available and sample or experimental information (Z) obtained to predict the state of the world (θ) that will likely occur. Posterior probability $P(\theta_j|Z_k)$ is the probability of observing state of the world θ_j conditional on observing outcome Z_k from experiment $Z(k = 1, 2, \ldots, o), (Z_k \epsilon Z)$.

One advantage of the Bayesian approach to decision problems is that it is possible to modify $P(\theta)$ as additional information becomes available. The posterior probabilities $P(\theta|Z)$ reflect this modification. Bayes' theorem is used to combine prior and sample or experimental information. The posterior probability is calculated as shown in equation 2.1.

$$P(\theta_j|Z_k) = \frac{P(\theta_j)\ P(Z_k|\theta_j)}{P(Z_k)} \quad \text{where} \qquad 2.1$$

$$P(Z_k) = \sum_j P(\theta_j)\ P(Z_k|\theta_j) \qquad 2.1'$$

In equation 2.1, $P(Z_k|\theta_j)$ is the conditional probability of observing experiment outcome Z_k when θ_j is the true state of the world. $P(Z_k)$, in equation 2.1', is the marginal probability of observing the k-th outcome of experiment Z.

The only restrictions placed on the prior, conditional, marginal, and posterior probabilities (if defined) are that they be nonnegative and sum to unity. The restrictions are shown in equations 2.2.1 through 2.2.3. Failure to comply with these restrictions results in an inconsistency.

$$\sum_j P(\theta_j) = 1 \qquad j = 1, \ldots, n \qquad 2.2.1$$

$$\sum_k P(Z_k|\theta_j) = 1 \qquad k = 1, \ldots, o \qquad 2.2.2$$

$$\sum_k P(Z_k) = 1 \qquad k = 1, \ldots, o \qquad 2.2.3$$

B. Computing Bayesian Strategies

A Bayesian strategy is the selection of the act A_i that maximizes expected gains or weighted average gains for a given probability distribution over the states of the world. Two types of Bayesian strategies can be developed in a Bayesian decision model. One utilizes the prior probability distribution $P(\theta)$ and the other utilizes the posterior probability distribution $P(\theta|Z)$. The strategy developed using the prior probability distribution is commonly referred to as the "no data" problem, and the strategy developed using the posterior probability distribution is referred to as the "data" problem. The latter term implies use of additional sample information or "data."

Utilizing the prior probability distribution $P(\theta)$, the expected gain of act A_i, $EG'(A_i)$, is computed as shown in equation 2.3.

$$EG'(A_i) = \sum_j g_{ij} P(\theta_j) \tag{2.3}$$

Similarly, the expected gain of act A_i utilizing the posterior probability distribution given that Z_k is the experimental outcome is computed as shown in equation 2.4.

$$EG''(A_{ik}) = \sum_j g_{ij} P(\theta_j|Z_k) \tag{2.4}$$

The single prime used in equation 2.3 indicates the expected gain is determined utilizing the prior probability distribution; the double prime in equation 2.4 indicates the expected gain is calculated using the posterior distribution. The single and double prime notation will be used in the following subsection.

The Bayesian strategy utilizing $P(\theta)$ -- the no data strategy -- is to select act A_i' such that:

$$EG'(A_i') = \max_i EG'(A_i) \tag{2.5}$$

The Bayesian strategy for the "data" problem if the observed experimental outcome turns out to be Z_k is A''_{ik} where:

$$EG''(A''_{ik}) = \max_i EG''(A_{ik}) \qquad 2.6$$

The values of 2.6 can be computed before the experimental outcome is observed. After the experimental outcome is observed, the data strategy is selected. It is simply the act identified in 2.6 as yielding maximum expected gain for the observed experimental outcome. Suppose the observed outcome is Z_g. Then the data strategy is A''_{ig} where

$$EG''(A''_{ig}) = \max_i EG''(A_{ig}) \qquad 2.6'$$

The computational procedures required to obtain a data Bayesian strategy are illustrated in table 2.2 assuming three actions are available and three states of the world are possible. The gains (g_{ij}) associated with each combination $A_i\theta_j$ are shown in the lower left corner of the table. The conditional probability of observing the k-th forecast when the true state of the world is θ_j, $P(Z_k|\theta_j)$ appears in the upper left corner of the table.

The posterior probability $P(\theta_j|Z_k)$ is obtained by use of equation 2.1. First, the priors $P(\theta_j)$ are multiplied by the conditional probability of a forecast state of the world given the true state of the world $P(Z_k|\theta_j)$. The resulting joint probabilities appear in the upper right corner of the table. The probability of observing the Z_k-th outcome or forecast is $P(Z_k)$ where k = 1, 2, or 3.

The second step in obtaining the posterior probabilities $P(\theta_j|Z_k)$ is to normalize the joint probabilities, $P(\theta_j)$ $P(Z_k|\theta_j)$, by dividing by

Table 2.2. Tabular form for computing the data Bayesian strategy for a three action, three state of the world problem

State of the world	$P(Z_k \mid \theta_j)$			Prior $P(\theta_j)$	$P(\theta_j)\ P(Z_k \mid \theta_j)$		
	Z_1	Z_2	Z_3		Z_1	Z_2	Z_3
θ_1	$P(Z_1 \mid \theta_1)$	$P(Z_2 \mid \theta_1)$	$P(Z_3 \mid \theta_1)$	$P(\theta_1)$	$P(\theta_1)\ P(Z_1 \mid \theta_1)$	$P(\theta_1)\ P(Z_2 \mid \theta_1)$	$P(\theta_1)\ P(Z_3 \mid \theta_1)$
θ_2	$P(Z_1 \mid \theta_2)$	$P(Z_2 \mid \theta_2)$	$P(Z_3 \mid \theta_2)$	$P(\theta_2)$	$P(\theta_2)\ P(Z_1 \mid \theta_2)$	$P(\theta_2)\ P(Z_2 \mid \theta_2)$	$P(\theta_2)\ P(Z_3 \mid \theta_2)$
θ_3	$P(Z_1 \mid \theta_3)$	$P(Z_2 \mid \theta_3)$	$P(Z_3 \mid \theta_3)$	$P(\theta_3)$	$P(\theta_3)\ P(Z_1 \mid \theta_3)$	$P(\theta_3)\ P(Z_2 \mid \theta_3)$	$P(\theta_3)\ P(Z_3 \mid \theta_3)$
					$P(Z_1)$	$P(Z_2)$	$P(Z_3)$

Action	State of the world θ_j			State of the world	Posterior probability $P(\theta \mid Z)$		
A_i	θ_1	θ_2	θ_3	θ_j	Z_1	Z_2	Z_3
A_1	g_{11}	g_{12}	g_{13}	θ_1	$P(\theta_1 \mid Z_1)$	$P(\theta_1 \mid Z_2)$	$P(\theta_1 \mid Z_3)$
A_2	g_{21}	g_{22}	g_{23}	θ_2	$P(\theta_2 \mid Z_1)$	$P(\theta_2 \mid Z_2)$	$P(\theta_2 \mid Z_3)$
A_3	g_{31}	g_{32}	g_{33}	θ_3	$P(\theta_3 \mid Z_1)$	$P(\theta_3 \mid Z_2)$	$P(\theta_3 \mid Z_3)$

Action	$EG''(A_{ik})$		
A_1	$EG''(A_{11})$	$EG''(A_{12})$	$EG''(A_{13})$
A_2	$EG''(A_{21})$	$EG''(A_{22})$	$EG''(A_{23})$
A_3	$EG''(A_{31})$	$EG''(A_{32})$	$EG''(A_{33})$
Maximizing action	$EG''(A''_{i1})$	$EG''(A''_{i2})$	$EG''(A''_{i3})$

$P(Z_k)$. The posterior probabilities are shown in the center section of the right hand side of table 2.2.

The expected gain of selecting action i after observing the k-th forecast or sample results, $EG''(A_{ik})$ is determined by use of equation 2.4.

The "data" Bayesian strategy is to select the action (A''_{ik}) that maximizes the expected gain after observing the k-th experimental results. The maximizing action is illustrated in the bottom right hand line of table 2.2 for each of the k experiment results.

The "no data" Bayesian strategy for table 2.2 is computed by use of equations 2.3' and 2.5'.

$$EG'(A_i) = \sum_{j=1}^{3} g_{ij} \; P(\theta_j) \text{ for all } A_i \text{ in } A \qquad 2.3'$$

The Bayesian "no data" strategy is the action that maximizes expected gains:

$$EG'(A_i') = \max_i \; EG'(A_i) \qquad 2.5'$$

A_i' is the best the decision maker can do using $P(\theta)$.

Advantages of the Bayesian strategy, as listed by Halter and Dean (19, p. 120), are:

(1) Bayesian strategies corresponding to all sets of prior probabilities contain all admissible strategies.

(2) A Bayesian strategy is a pure strategy.

(3) Bayesian strategies are relatively easy to compute.

C. Values of Information

It has been assumed that the decision maker is confronted with the problem of decision making under uncertainty. Although the outcome of a decision is not known with complete certainty, the parameters of the probability distribution are known with certainty, hence, a known probability for each outcome has been established.

A decision made on the basis of the current state of information available, whether it be a "prior" or "posterior" probability distribution of states of the world, is a "terminal" decision. If the opportunity exists to collect more sample information before making a "terminal" decision, the decision maker is confronted with a "preposterior" decision. The name "preposterior" is applicable because the option exists of obtaining additional sample information (21, p. 551).

It may be of interest to the decision maker to determine the economic value of additional information before an experiment is performed or a subscription fee is paid for the forecast information. If the value of information exceeds its cost, then the additional information should probably be obtained, otherwise not.

1. Value of perfect information

The most valuable information one could obtain is "perfect" information. Perfect information reduces the decision to one of certainty rather than uncertainty. If it is known that θ_j will occur, the decision maker selects act A_g such that:

$$G(A_g\theta_j) = \max_i G(A_i\theta_j) \qquad 2.7$$

A_g is the act with the maximum gain. Equation 2.7 is similar to equation 2.5'. In equation 2.7 the payoff is known with certainty. The maximum expected payoff or gain is used as the decision criteria in equation 2.5'.

The value of perfect information is conditional on the observed state of the world θ_j and is commonly referred to as "conditional value of perfect information," (CVPI) (36, p. 88). CVPI is calculated as shown in equation 2.8.

$$\text{CVPI}|\theta_j = G(A_g\theta_j) - G(A_i'\theta_j) \qquad 2.8$$

A_i' is the act defined in equation 2.5 that maximizes expected gain under the decision maker's prior distribution of θ. The CVPI is always nonnegative.

Because θ_j has already occurred, CVPI$|\theta_j$ is of questionable value to the decision maker. It is, however, possible to determine the "expected" value of perfect information (EVPI) prior to observing some element from θ. EVPI is calculated as shown in equation 2.9.

$$\text{EVPI} = \sum_j P(\theta_j)\ (\text{CVPI}|\theta_j) \qquad 2.9$$

Each CVPI$|\theta_j$ is weighted by the decision maker's prior probability of observing θ_j. EVPI is the maximum amount the decision maker should be willing to pay for perfect information.

2. Value of sample information

Once the Bayesian strategies have been selected for the no data problem [EG'(A_i')] and determined for each of the possible sample results of experiment Z $[EG''(A''_{ik})]$, the conditional and expected value of sample information can be computed. Both CVSI and EVSI are unique to the specific experiment under consideration. EVPI is independent of additional sample information.

CVSI is calculated in a manner similar to CVPI. A' was defined as the optimal act under the prior distribution (equation 2.5) and A''_{ik} as the optimal act under the posterior distribution (equation 2.6) determined after observing outcome k of some real experiment Z. If the decision maker performs experiment Z, observes k, and selects A''_{ik} (data strategy) rather than A'_i (no data strategy), his expected terminal gain or payoff has been increased by the amount determined in equation 2.10.

$$CVSI|Z_k = EG''(A''_{ik}) - EG''(A'_i) \qquad 2.10$$

Note that CVSI is conditional on observing the k-th outcome of experiment Z, and that the expected gain of act A'_i is determined using the posterior probability distribution for θ rather than the prior as shown in equations 2.3 and 2.5.

EVSI can be calculated prior to observing sample or experiment results as shown in equation 2.11. CVSI for each possible sample result is weighted by the marginal probability $[P(Z_k)]$ of observing each outcome. $P(Z_k)$ is calculated as shown in equation 2.1'.

$$EVSI = \sum_k P(Z_k)\ (CVSI|Z_k) \qquad 2.11$$

EVSI is the weighted average gain that can be expected above the previously "terminal" decision if the data strategy is followed for an extended period of time. Similar to EVPI, EVSI can never be less than zero.

3. Expected net gain of sample information

The expected net gain of sampling (ENGS) or experimenting is defined as the difference between the expected value of sample information (EVSI, equation 2.11) and the cost of obtaining the sample (CS) (21, p. 562; 36, p. 91).

$$\text{ENGS} = \text{EVSI} - \text{CS} \qquad 2.12$$

D. Decision Rules and Utility

The Bayesian strategy, the action with the maximum expected gain or payoff measured in dollars, may have some pitfalls. One pitfall can be illustrated by the two action - two states of the world problem shown below.

	States		
Acts	θ_1	θ_2	Expected value
A_1	\$ 0	\$ 0	\$ 0
A_2	50,000	-40,000	5,000
$P(\theta_j)$	1/2	1/2	

A decision maker confronted with this problem and following a no data Bayesian strategy would select act A_2 since its expected payoff is

\$5,000 versus zero payoff for act A_1. Almost any decision maker would be willing to accept a payoff of \$50,000 if θ_1 were to occur; however, a relative few could accept the \$40,000 loss if θ_2 were to occur (this is particularly true for graduate students).

The possibilities of incurring a sizeable loss may influence a decision maker to select a non-Bayesian strategy or use another criterion such as those used in game theoretic models. This implies that the payoffs need to be redefined in terms that reflect the subjective attitude of the decision maker towards various gains and losses. An economic term often used to describe this subjective satisfaction is "utility." It is possible to express the monetary consequences of each combination $A_i\theta_j$ in terms of utility via a real function[1] if the real function is known or can be determined.

U(M) is the utility associated with consequence or payoff level M. The utility function maps U(M) for every M. The properties of a utility function described by Chernoff and Moses (12, p. 81) are:

(1) $U(M_1) > U(M_2)$ if and only if the decision maker prefers M_1 to M_2.

(2) If M is the prospect where, with probability θ, the decision maker faces M_1 and with probability $1 - \theta$ he faces M_2, then $U(M) = \theta U(M_1) + (1 - \theta)\, U(M_2)$.

The first property states that utility increases when the prospect improves. The second property states that utility can be computed with ordinary odds. To illustrate the second property Chernoff and Moses

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[1]If two variables M and U are related so that, for each M in a domain R of real numbers, we obtain one or more real values for U, then U is said to be a real function of the real variable M defined in the domain R (33, p. 19).

(12, p. 96) identify four expectation properties as:

(1) $E(X+Y) = E(X) + E(Y)$

(2) $E(cX) = cE(X)$

(3) $E(1) = 1$

(4a) $E(X) > E(Y)$ if $X > Y$

(4b) $E(X) \geq E(Y)$ if $X \geq Y$

Expectation properties 1 and 2 are combined to form the second property of a utility function shown above. This indicates that the long run expected utility of prospects M_1 and M_2 is $U(M)$.

The basic assumptions necessary for the existence of a utility function discussed by Chernoff and Moses (12, p. 82) are:

(1) With sufficient calculation a decision maker faced with two prospects M_1 and M_2 will be able to decide whether he prefers prospect M_1 to M_2, whether he likes each equally well, or whether he prefers M_2 to M_1.

(2) If M_1 is regarded at least as well as M_2, and M_2 at least as well as M_3, then M_1 is regarded at least as well as M_3.

(3) If M_1 is preferred to M_2 which is preferred to M_3, then there is a mixture of M_1 and M_3 which is preferred to M_2, and there is a mixture of M_1 and M_3 over which M_2 is preferred.

(4) If the decision maker prefers M_1 to M_2, and M_3 is another prospect, then the decision maker will prefer a mixture of M_1 and M_3 to the same mixture of M_2 and M_3.

These four assumptions are expressed in terms of ranking, transitivity, semi-strict convexity or continuity, and independence of irrelevant alternatives, respectively, by Halter and Dean (19, p. 50), Henderson and Quandt (24a, p. 13), and Arrow and Hahn (2, p. 82).

The decision maker's attitude towards uncertainty will be reflected by the shape of his utility function with respect to monetary gains. Three basic shapes are possible and are illustrated in figure 2.1. Utility function I indicates that the decision maker is indifferent towards uncertainty. Maximizing expected monetary values will maximize expected utility. This occurs whenever the utility function is linear ($\frac{d^2u}{dm^2} = 0$). The decision maker is said to be indifferent towards risk. The prospect of gaining each additional dollar adds a constant amount to total satisfaction.

A utility function similar to II indicates that the decision maker is a risk averter. The possibility of gaining each additional dollar increases total utility less than the previous dollar increased utility ($\frac{d^2u}{dm^2} < 0$). Utility function III is characteristic of a risk taker.

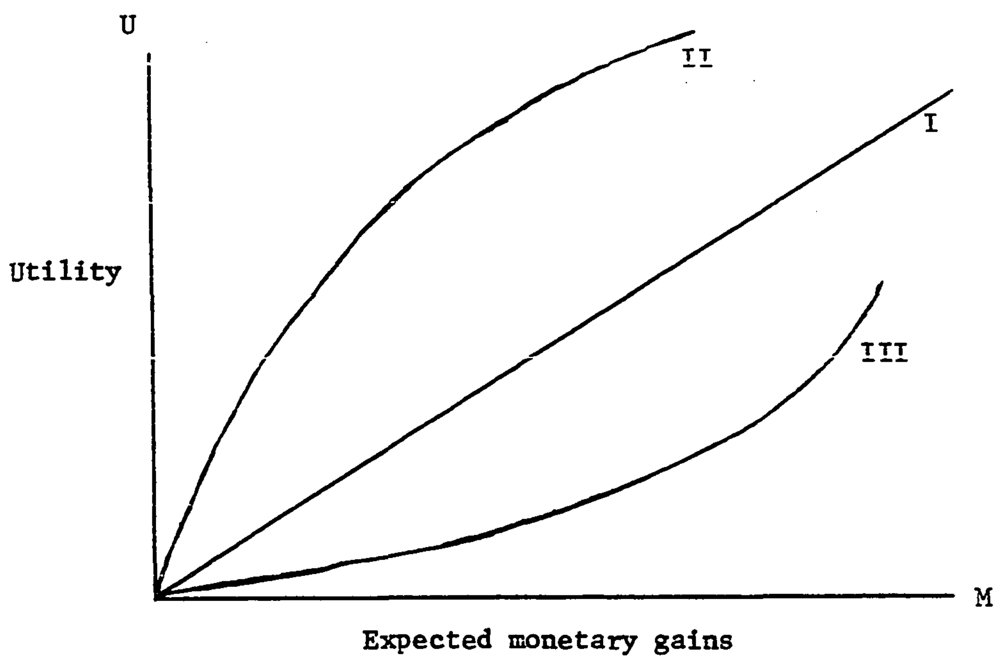


Figure 2.1. Shape of the utility function as influenced by attitudes toward risk

The function indicates that the possibility of higher monetary gains increases total utility at an increasing rate ($\frac{d^2u}{dm} > 0$). It may be that the utility function of a decision maker will assume all three shapes over the domain.

Raiffa and Schlaifer state that the objective of mathematical analysis of decision making under uncertainty is to identify a course of action (which may or may not include experimentation) that is logically consistent with the decision maker's own preferences for consequences, and that these consequences can be expressed by numerical utilities (36, p. vii). It is not the purpose of this section to show how the numerical utilities are determined but only to illustrate that they in fact do influence selection of "optimal" courses of action. For readings concerning techniques used in determining utility functions, the reader is referred to Halter and Dean (19, ch. 3), and Officer and Halter (32).

It was assumed for this dissertation that the decision maker (cattle feeder) is indifferent toward uncertainty; hence, his utility function is linear with respect to money over the relevant range. Using monetary payoffs will result in selecting the same strategies as using utility payoffs if a decision maker's utility function for monetary gains is linear (21, p. 535; 9, p. 6; 19, p. 46).

E. Review of Selected Studies Utilizing Bayesian Decision Models

Eidman, Carter, and Dean provide an empirical application of Bayesian decision theory to turkey production (15). The problem is one of choosing between contract and independent production of turkeys using both prior and posterior probabilities. The states of world considered are product prices and turkey mortality rates. The value of additional information provided by the price forecasting model is substantial.

Carlson utilizes a Bayesian decision model to arrive at optimal crop disease control practices for California peach growers in controlling peach brown-rot (11). Optimal pesticide use actions are computed for three different objective functions -- maximum subjective expected returns, mean-variance of returns, and maximum expected returns with a minimum income side condition.

Possible applications of Bayesian decision theory in forest management are presented by Thompson (43). The first area of application is to determine whether or not to prune trees. Random variables (states of the world) are: (1) price of timber, (2) yield, and (3) interest rates. The second area of application is to determine the optimal size of fire crew to maintain. Size of fire in terms of acreages burned per day is the random variable used.

The usefulness of Bayesian decision theory concerning freeze protection in production of citrus crops is illustrated by Sporleader (40). A procedure for integrating temperature probability estimates into an analysis of decisions under uncertainty is examined, and this reduces the problem to one of risk.

Leath used a Bayesian decision model to develop a method for making stocking rate decisions for Oklahoma small grain pastures (25). In the model, weather is the random variable.

Bullock and Logan utilize Bayesian decision theory in developing a set of decision criteria to assist cattle feeders in making purchasing and marketing decisions when faced with uncertainty about future cattle prices (8, 9). Decision criteria are developed for the following four models:

(1) A direct application of Bayesian decision theory to determine the minimum expected price change required to induce feeding a particular lot of cattle another 30 days.

(2) The minimum expected price change necessary to induce feeding cattle another 60 days is determined. This is an extension of the first model. The model is applicable only if a sell decision is generated from model one and cattle weigh less than 1,000 pounds.

(3) A set of buy-or-not-buy decision criteria are developed for feeder cattle based on expected feeding margins.

(4) Results of the first three models are incorporated into a simulation model. The model simulates buying, feeding, and selling activities six months into the future.

All decision criteria are based on slaughter and feeder cattle price expectations. Considerations which allow for change in feed prices and/or ration costs are not incorporated into the model. Likewise, only one "typical" ration and one rate of gain is considered for each weight group of cattle.

III. THE MODEL

A. Introduction

Profits derived from a cattle feeding enterprise are attributed to several variables:

$$\pi = f(P^*_{ij}, F^*_{ij}, P^{**}_{ij}, F^{**}_{ij}, Q_f, g, T, R, VC, FC) \qquad 3.1$$

where

π = profits

P^*_{ij} = price received per pound for cattle of i-th weight and j-th quality grade

F^*_{ij} = proportion of cattle of i-th weight and j-th quality grade sold

P^{**}_{ij} = purchase price per pound for feeder cattle of i-th weight and j-th quality grade

F^{**}_{ij} = proportion of cattle of i-th weight and j-th quality grade purchased

Q_f = initial weight of feeder cattle

g = pounds of gain per animal per day

T = number of days the animal is retained on feed

R = ration costs per day

VC = variable costs other than R such as veterinary and medical expenses, fuel, repairs, labor, etc.

FC = fixed costs

Generally when the decision is made to purchase feeder cattle, the entrepreneur knows current cattle prices (P^{**}_{ij}), variable and fixed costs (VC, FC), and has some idea of the grade distribution of feeder and slaughter cattle (F_{ij}^{**}, F_{ij}^{*}) based on previous feeding experience. Given future price expectations $E(P_{ij}^{*})$ the entrepreneur must decide what daily rate of gain to feed for (g), the ration to feed, and how long to retain the cattle on feed (T).

At the time cattle are placed in the feedlot the operator has some idea as to what future market conditions (price) will be or at least what price is required to break even. This price expectation can be based on either objective or subjective sources. As indicated in chapter one, once cattle have been placed on feed the consequences of the feeding enterprise are a function of changes in fed cattle prices. Price increases or decreases, increase or decrease profits, respectively, and may affect other variables that influence profits such as T, g, and R.

Two variables that directly influence the profitability of the cattle feeding enterprise are the daily rate of gain (g) and number of days fed (T). The two variables are not necessarily independent of each other. Assuming that an animal is to be marketed within a given weight range, then the greater the daily gain the shorter the required feeding period and vice versa.

For any feeder animal, the daily rate of gain (g) is a function of the energy in the feed ration being consumed. Also, a given rate of gain can be obtained from more than one combination of feed ingredients. A change in price of one or more feeds may affect the combination of ingredients and costs of the least cost ration used to attain a specific rate of gain. For any given set of feed prices it will be shown in section E of this chapter that higher rates of gain can be attained only with more expensive rations, hence total ration costs are increased.

The cattle feeder who attempts to maximize net revenue will try to attain that rate of gain for which the difference between total revenue and total costs is the greatest. This, of course, is that rate of gain where marginal cost is equal to marginal revenue of the beef produced. Hence, the profit maximizing rate of gain is a function of not only expected beef prices but also feed ingredient prices.

The time variable (T) affects profits in three ways. First, an increase in T increases total costs due to additional feed consumed, additional expenses attributed to labor, medical and veterinary bills, etc. Second, a higher proportion of the total feed consumed during the feeding period is required for body maintenance. This leaves a smaller proportion of total feed consumed available for growth, hence, the marginal daily rate of gain decreases for an animal fed a constant ration. Third, assuming a homogenous group of cattle started on feed at an equal age and weight, and gaining an equal amount per day, then as T increases, so do age and weight, and the proportion of cattle attaining a higher quality grade also increases. There is, however, an upper limit on the age at which an animal can grade good or higher. Under both the existing and proposed grade standards[1], it is not

[1] On September 10, 1974, the USDA announced a proposal to revise the U.S. standards for grades of beef. Details of the changes can be found in the Federal Register of September 11, 1974 (34). At the date of this writing the proposed system had not been adopted, therefore, the decision was made to base the analyses on data concerning grades, price relationships, etc., using grading standards existing at the time of the proposed revision. If and when the proposed standards are implemented and sample data becomes available, the model used in this dissertation could be updated.

possible for an animal to grade higher than commercial if older than 48 and 42 months, respectively (37, p. 3). The weighted average price received increases as average quality grade increases.

Similar to the marginal conditions associated with g, cattle should be retained in the feedlot to the point in time that marginal revenue equals marginal cost of retaining cattle an additional day if profits are to be maximized.

One purpose of this dissertation is to develop an economic model that will provide the cattle feeder a means to simultaneously determine the daily rate of gain, associated least cost ration, and length of time to retain cattle on feed to maximize expected profits when feed ingredient prices are known and livestock price expectations established.

B. Review of Related Cattle Feeding Studies

Faris presents appropriate economic techniques and criteria to use in determining when to replace cattle currently in the feedlot to maximize revenue over time (16). The economic criterion developed by Faris is based on the assumptions that the decision maker attempts to maximize average net revenue with respect to time versus maximizing net revenue from each pen of cattle and that all prices are known with certainty. The Faris study did not allow for varying rates of gain or rations. Faris concluded that the optimum time to replace cattle is when the marginal net revenue from the present group of cattle equals the maximum average net revenue anticipated from the subsequent lot of cattle.

Nelson and Purcell extend the economic criteria developed by Faris to determine the optimal time to replace cattle (29). A comparison of optimal time to sell cattle is made using marginal net revenue associated with liveweight, carcass weight, and lean meat weight as criterion. As in the Faris study, no consideration is given to uncertainty in prices or to variation in rates of gain. Both of the latter two studies emphasized the "time" element.

The feasibility of a beef feedlot information system is explored by Nelson (28). The study is an adaptive multi-period statistical decision model for the analysis of relevant data to formulate information regarding the best course of action. The information system is divided into three subsystems: (1) a cattle price forecasting subsystem that determines the mathematical expectation of future monthly prices, (2) a beef feed formulation subsystem to determine the minimum monthly cost of feeding cattle of different weights for various rates of gain utilizing TDN as a basis for ration formulation, and (3) a feedlot operations scheduling subsystem that uses a dynamic programming algorithm to determine the optimal buying, selling, and feeding decisions for the current month.

Although reviewed in the previous chapter, the author feels that the work by Bullock and Logan (8, 9) concerning cattle feedlot marketing decisions under uncertainty should again be referenced.

C. Overview of the Model

The two variables g and T provide several feeding alternatives. Each combination of g and T represents one feeding alternative or strategy available to the cattle feeder. Cattle can be fed one ration through the entire feeding program or various rations and for different rates of daily gain as feed prices and livestock price expectations fluctuate.

The planning horizon for any group of cattle can be divided into decision intervals or production periods. The planning horizon extends from the time cattle are purchased or planned to be purchased until the latest time that cattle can be retained in the feedlot or until maximum marketable weight is reached, whichever comes first. A decision interval and/or production period is defined as the period of time cattle are retained on a specific ration or until the existing strategy or feeding alternative is re-evaluated. For illustrative purposes the conceptual decisions, decision intervals, and strategies available to the cattle feeder are illustrated in figure 3.1.

Figure 3.1 illustrates a planning horizon containing three decision intervals. Each decision interval contains elements for decision making, denoted by the rectangular boxes, and activities, denoted by circles. A_{abc} represents a specific combination of feeding activities during the decision interval denoted by the last nonzero a, b, or c subscript and during each preceding interval. A_{abcs} is a selling activity corresponding to the A_{abc}-th feeding activities. The subscripts a, b, and c represent the feeding level or rate of gain expected during each decision interval. A_{230} is a feeding action during the second decision interval for cattle that have been fed at rate of gain two during the first decision interval and are fed at

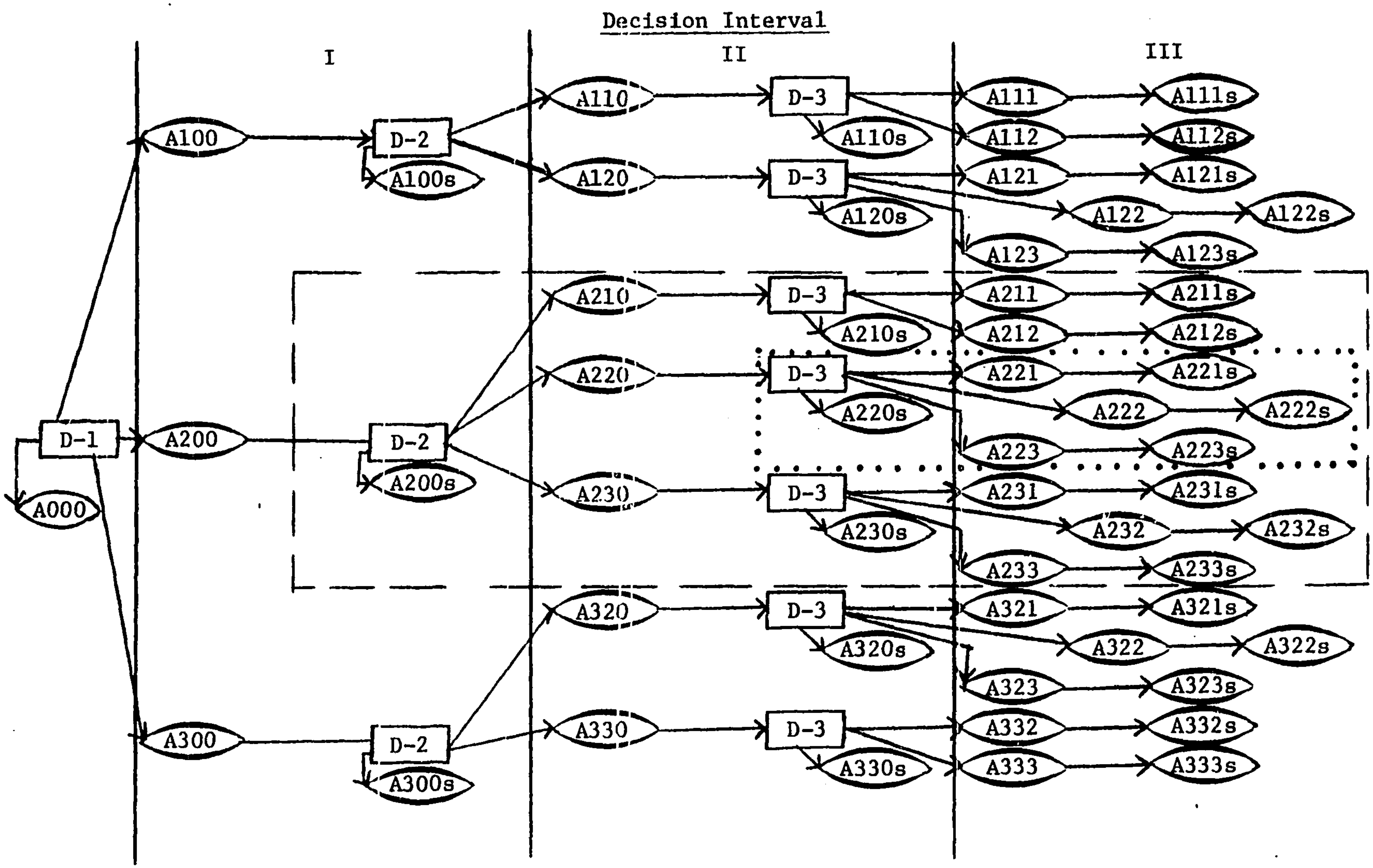
Figure 3.1. A decision tree for a feeding program involving three decision intervals and three rates of gain

Legend:

decision node

feeding strategy

→ direction of possible movement


Decision Interval
I
II
III
D-1
A000
A100
A200
A300
D-2
A100s
A200s
A300s
A110
A120
A210
A220
A230
A320
A330
D-3
A110s
A120s
A210s
A220s
A230s
A320s
A330s
A111
A112
A121
A122
A123
A211
A212
A221
A222
A223
A231
A232
A233
A321
A322
A323
A332
A333
A111s
A112s
A121s
A122s
A123s
A211s
A212s
A221s
A222s
A223s
A231s
A232s
A233s
A321s
A322s
A323s
A332s
A333s

rate of gain three during the second decision interval. A_{230s} is the sell activity associated with A_{230}.

In figure 3.1 the subscripts a, b, and c can vary as follows:

$a = 0, 1, 2, 3$

$0 \leq b \leq 3;\ a - 1 \leq b \leq a + 1$

$0 \leq c \leq 3;\ b - 1 \leq c \leq b + 1$

An animal's rate of gain in any production period cannot differ from his rate of gain in the previous period by more than one rate of gain. The only exception is in the first production period where one of three possible rates can be attained.

At D-1 the decision maker must determine which of the 28 feeding alternatives maximize expected profits. An economic decision model was used to select the feeding strategy that maximized expected profits. If none of the feeding activities appear profitable, then A_{000} is selected; this is a decision not to feed cattle. For example, suppose that for the feed prices and livestock price expectations existing at D-1, A_{100s}, A_{230s}, and A_{333s} maximize expected discounted returns for the first, second, and third decision intervals, respectively, and returns from A_{100s} < returns from A_{333s} < returns from A_{230s}. Then the decision maker selects activity A_{200} for the first production period. Activity A_{200} is a prerequisite for activity A_{230}.

At the end of the first decision interval or production period, at node D-2, the decision maker re-evaluates earlier decisions as influenced by new price information and expectations. The decision is made to either sell or continue feeding the cattle.

All strategies to be considered at D-2 are enclosed in the outer boxed area (dashed line) of figure 3.1. If total profits cannot be increased or losses decreased by continued feeding, the cattle will be sold (A_{200s}).

Assuming that A_{210s} and A_{223s} increase profits for the next two decision intervals, respectively, and expected profits from A_{223s} exceed those from A_{210s}, then A_{220} is the selected feeding action for decision interval two. A_{220} is the prerequisite for A_{223s}.

At the end of the second decision interval, at node D-3, the decision again must be made to either sell the cattle or continue feeding. Updated price information and/or expectations can be included. Assuming the cattle must be sold at the end of the third decision interval, the four strategies available are contained in the inner box (dotted lines of figure 3.1 (A_{220s}, A_{231s}, A_{222s}, A_{223s}). If total profits can be increased or losses minimized, the cattle will be continued on feed, otherwise the sell action (A_{220s}) is implemented.

The economic decision model used to select the optimal feeding and marketing strategy in figure 3.1 incorporated several production and marketing variables. A summary of required input data, analyses, and output information of the economic decision model are shown in figure 3.2. A linear programming model was developed to calculate the least cost ration for each feeding alternative as a function of average weight of the animal, expected daily rate of gain, and feed ingredient prices.

Return above variable costs for each feeding alternative was calculated by subtracting expense of the least cost ration, purchase

Figure 3.2. Schema of data, analyses, and output of an economic livestock feeding-marketing decision model

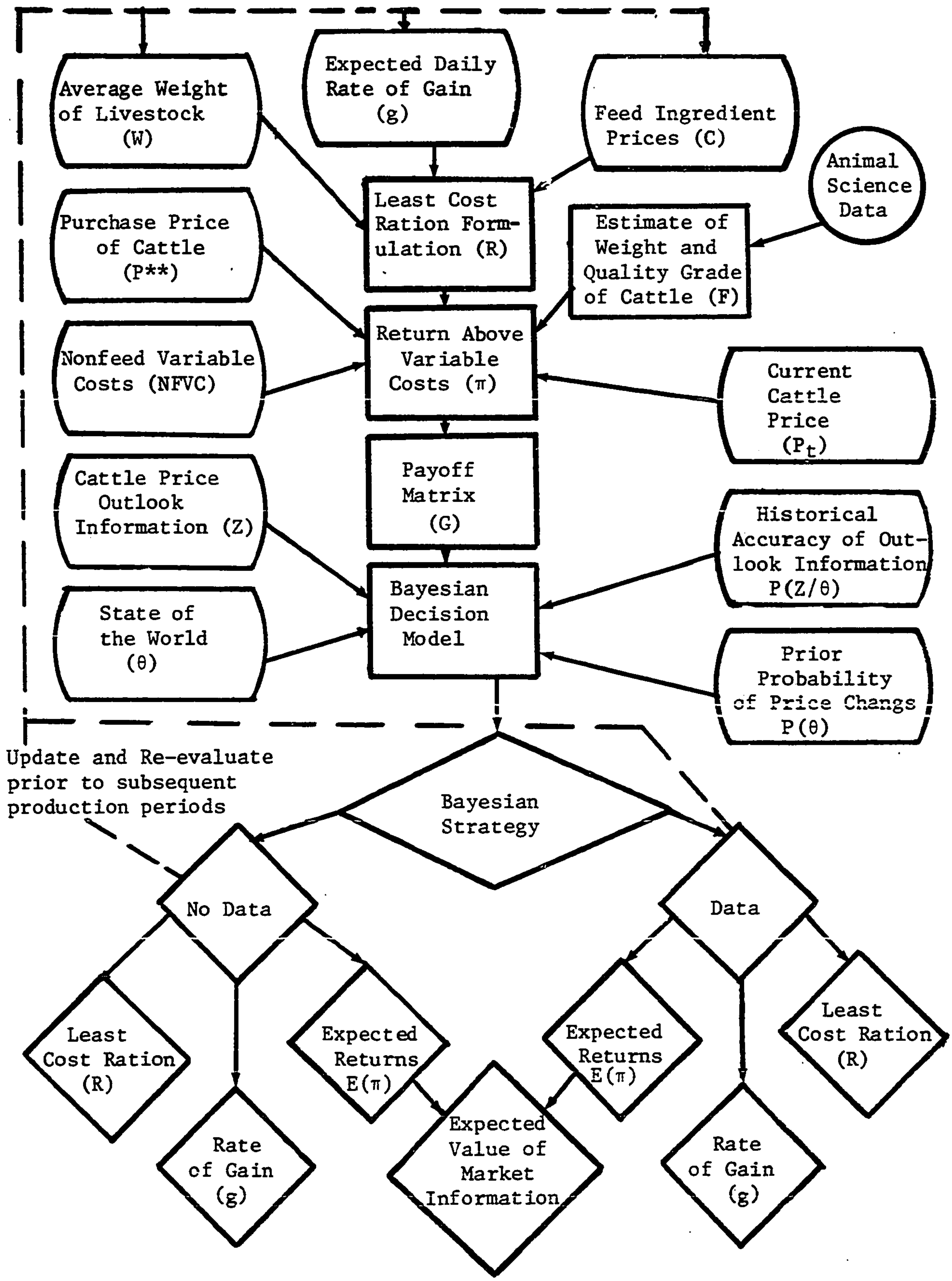
Legend:

○ input

□ economic model

◇ output

( ) data



Average Weight of Livestock (W)
Expected Daily Rate of Gain (g)
Feed Ingredient Prices (C)
Animal Science Data
Least Cost Ration Formulation (R)
Purchase Price of Cattle (P**)
Estimate of Weight and Quality Grade of Cattle (F)
Nonfeed Variable Costs (NFVC)
Return Above Variable Costs (π)
Current Cattle Price (P_t)
Payoff Matrix (G)
Cattle Price Outlook Information (Z)
Historical Accuracy of Outlook Information P(Z/θ)
Bayesian Decision Model
State of the World (θ)
Prior Probability of Price Change P(θ)
Update and Re-evaluate prior to subsequent production periods
Bayesian Strategy
No Data
Data
Least Cost Ration (R)
Rate of Gain (g)
Expected Returns E(π)
Expected Value of Market Information
Expected Returns E(π)
Rate of Gain (g)
Least Cost Ration (R)

cost of the animal, and other nonfeed variable costs from total revenue. Total revenue was computed as a function of current cattle prices and expected weight and quality grade of the cattle at marketing. Each feeding alternative represents one action in the Bayesian decision model.

As discussed in chapter two, other required components of a Bayesian decision model are states of the world, and a prior probability distribution of states of the world if a no data strategy is obtained; in addition, cattle price forecast information, and the conditional probability of the forecast state of the world given the true state of the world for a data strategy. States of the world are changes in choice slaughter cattle prices from current levels. Cattle price forecasts were obtained from the Iowa Farm Outlook Letter.

The results of the decision model provide both the no data and data Bayesian cattle feeding-marketing strategies. In addition, the least cost ration, profit maximizing daily rate of gain, optimal time to retain cattle on feed, and expected returns are determined for each Bayesian strategy. The difference between expected income from following the no data strategy and expected weighted average income from following the data strategy is the expected value of slaughter cattle market information in the Iowa Farm Outlook letter.

The Bayesian strategies are re-determined prior to each subsequent decision interval or production period.

D. Assumptions of the Cattle Feeding Program

Following is a list of basic assumptions used concerning the cattle feeding program considered in this dissertation.

1. The 240 day planning horizon consists of four 60 day decision intervals. At the end of each decision interval feed prices and livestock price expectations were updated to include all available information.

2. Expected animal growth and performance is consistent with standards published by the National Academy of Science (42, p. 3). Four rates of gain considered were 1.5, 2.0, 2.5, and 3.0 pounds per day, respectively. The feeding alternatives considered are shown in table 3.1. The subscript notation associated with each A_{abcd} is similar by definition to the notation used in the previous section; however, one additional decision interval is considered (d-th) and one additional rate of gain is feasible. The restrictions placed on the subscripts are as follows:

$a = 0, 1, 2, 3, 4$

$0 \leq b \leq 4;\ a - 1 \leq b \leq a + 1$

$0 \leq c \leq 4;\ b - 1 \leq c \leq b + 1$

$0 \leq d \leq 4;\ c - 1 \leq d \leq c + 1$

Associated with each feeding activity (A_{abcd}) is a possible selling action (A_{abcds}). The selling actions are not shown but are implied in each A. In determining weight relationships between feeding activities, it is assumed that yearling steers were purchased weighing 750 pounds. A seven percent inshrink and four percent outshrink are charged when determining initial weight at the feedlot and final pay weight, respectively. The initial weight, mean weight, and final weight associated with each A_{abcd} are shown in table 3.1.

Table 3.1. Cattle feeding alternatives considered assuming four 60 day decision intervals and four rates of gain in the Bayesian decision model

Activity	Initial weight	Average daily gain for last 60 days	Ending weight	Mean weight during feeding period	Accumulated days on feed
		pounds			days
A_{0000}	--	--	--	--	--
A_{1000}	698	1.5	788	743	60
A_{2000}	698	2.0	818	758	60
A_{3000}	698	2.5	848	773	60
A_{4000}	698	3.0	878	788	60
A_{1100}	788	1.5	878	833	120
A_{1200}	788	2.0	908	848	120
A_{2100}	818	1.5	908	863	120
A_{2200}	818	2.0	938	878	120
A_{2300}	818	2.5	968	893	120
A_{3200}	848	2.0	968	908	120
A_{3300}	848	2.5	998	923	120
A_{3400}	848	3.0	1028	938	120
A_{4300}	878	2.5	1028	953	120
A_{4400}	878	3.0	1058	968	120
A_{1110}	878	1.5	968	923	180
A_{1120}	878	2.0	998	938	180
A_{1210}	908	1.5	998	953	180
A_{1220}	908	2.0	1028	968	180
A_{1230}	908	2.5	1058	983	180

Table 3.1. Continued

Activity	Initial weight	Average daily gain for last 60 days	Ending weight	Mean weight during feeding period	Accumulated days on feed
			pounds		days
A_{2110}	908	1.5	998	953	180
A_{2120}	908	2.0	1028	968	180
A_{2210}	938	1.5	1028	983	180
A_{2220}	938	2.0	1058	998	180
A_{2230}	938	2.5	1088	1013	180
A_{2320}	968	2.0	1088	1028	180
A_{2330}	968	2.5	1118	1043	180
A_{2340}	968	3.0	1148	1058	180
A_{3210}	968	1.5	1058	1013	180
A_{3220}	968	2.0	1088	1028	180
A_{3230}	968	2.5	1118	1043	180
A_{3320}	998	2.0	1118	1058	180
A_{3330}	998	2.5	1148	1073	180
A_{3340}	998	3.0	1178	1088	180
A_{3430}	1028	2.5	1178	1103	180
A_{3440}	1028	3.0	1200	1114	178
A_{4320}	1028	2.0	1148	1088	180
A_{4330}	1028	2.5	1178	1103	180
A_{4340}	1028	3.0	1200	1114	178
A_{4430}	1058	2.5	1200	1129	177
A_{4440}	1058	3.0	1200	1129	168

Table 3.1. Continued

Activity	Initial weight	Average daily gain for last 60 days	Ending weight	Mean weight during feeding period	Accumulated days on feed
		pounds			days
A_{1111}	968	1.5	1058	1013	240
A_{1112}	968	2.0	1088	1028	240
A_{1121}	998	1.5	1088	1043	240
A_{1122}	998	2.0	1118	1058	240
A_{1123}	998	2.5	1148	1073	240
A_{1211}	998	1.5	1088	1043	240
A_{1212}	998	2.0	1118	1058	240
A_{1221}	1028	1.5	1118	1073	240
A_{1222}	1028	2.0	1148	1088	240
A_{1223}	1028	2.5	1178	1103	240
A_{1232}	1058	2.0	1178	1118	240
A_{1233}	1058	2.5	1200	1129	237
A_{1234}	1058	3.0	1200	1129	228
A_{2111}	998	1.5	1088	1043	240
A_{2112}	998	2.0	1118	1058	240
A_{2121}	1028	1.5	1118	1073	240
A_{2122}	1028	2.0	1148	1088	240
A_{2123}	1028	2.5	1178	1103	240
A_{2211}	1028	1.5	1118	1073	240
A_{2212}	1028	2.0	1148	1088	240
A_{2221}	1058	1.5	1148	1103	240

Table 3.1. Continued

Activity	Initial weight	Average daily gain for last 60 days	Ending weight	Mean weight during feeding period	Accumulated days on feed
		pounds			days
A_{2222}	1058	2.0	1178	1118	240
A_{2223}	1058	2.5	1200	1129	237
A_{2232}	1088	2.0	1200	1144	237
A_{2233}	1088	2.5	1200	1144	225
A_{2234}	1088	3.0	1200	1144	218
A_{2321}	1088	1.5	1178	1133	240
A_{2322}	1088	2.0	1200	1144	236
A_{2323}	1088	2.5	1200	1144	225
A_{2332}	1118	2.0	1200	1159	221
A_{2333}	1118	2.5	1200	1159	213
A_{2334}	1118	3.0	1200	1159	208
A_{2343}	1148	2.5	1200	1174	201
A_{3211}	1058	1.5	1148	1103	240
A_{3212}	1058	2.0	1178	1118	240
A_{3221}	1088	1.5	1178	1133	240
A_{3222}	1088	2.0	1200	1144	236
A_{3223}	1088	2.5	1200	1144	225
A_{3232}	1118	2.0	1200	1159	221
A_{3233}	1118	2.5	1200	1159	213
A_{3234}	1118	3.0	1200	1159	208
A_{3321}	1118	1.5	1200	1159	235

Table 3.1. Continued

Activity	Initial weight	Average daily gain for last 60 days	Ending weight	Mean weight during feeding period	Accumulated days on feed
		pounds			days
A_{3322}	1118	2.0	1200	1159	221
A_{3323}	1118	2.5	1200	1159	213
A_{3332}	1148	2.0	1200	1174	206
A_{3333}	1148	2.5	1200	1174	201
A_{4321}	1148	1.5	1200	1174	215
A_{4322}	1148	2.0	1200	1174	206

3. Sample data from the Allee Experiment Farm were used to determine the proportion of cattle grading choice or above at various weights. It is assumed that the type of cattle fed is comparable to the sample data. At the time of purchase approximately 25 percent of the steers grade choice and 75 percent grade good.

4. Cattle are marketed when expected short run profits are maximized for cattle weighing between 788 and 1200 pounds, when cattle weigh 1200 pounds, or at the end of the fourth decision interval, whichever occurs first. Cattle were not retained on feed after reaching 1200 pounds nor longer than 240 days. This criterion contrasts with optimal marketing strategies developed by Faris (16), and Nelson and Purcell (29). Criteria developed in these studies maximized long-run profits. Reasons for maximizing profits for a given group of cattle are:

a. The author's belief that the typical midwest farmer-cattle feeder feeds cattle as a means of marketing crops produced on the farm. This contrasts to the large, specialized commercial feedlots prevalent in the southwest and western sections of the country where the major portion of feed ingredients are purchased. The midwest farmer-cattle feeder attempts to maximize returns to his land base by marketing one or two groups of cattle per year. He is concerned with marketing strategies usually only when purchasing feeder cattle and selling finished slaughter animals.

b. A study by Purcell (35) revealed that 57 percent of Oklahoma cattle feeders surveyed try to maximize the return per head for each lot of cattle handled rather than maximize long-run profits.

Nineteen percent try to maximize returns to the total operation over some longer period of time.

E. Least Cost Rations

Developments in animal nutrition by Lofgreen and Garrett (26) have made traditional least cost linear programming models for ration formulation obsolete. The traditional models placed restrictions on total digestible nutrients, protein, and other selected nutrients. Heady (22, ch. 4), Beneke and Winterboer (3, ch. 12) illustrate linear programming models based on the traditional system.

1. Model formulation

The net energy (N.E.) system developed by Lofgreen and Garrett divides the net energy available in a ration into net energy available for maintenance and net energy available for growth (13, p. 610). Maintenance requirements are met prior to production or growth requirements. The net energy required for maintenance (NEm) is a function of body weight (W); net energy required for production or gain (NEg) is a function of the expected daily rate of gain (g) and body weight (W). The NEm and NEg requirements are expressed in equations 3.2 and 3.3, respectively (42, p. 3). Equation 3.3 expresses the NEg requirements for steers; heifers require a slightly higher energy ration to obtain comparable rates of gain.

$$\text{NEm} = 0.077W^{0.75} \qquad 3.2$$

$$\text{NEg} = (0.05272g + 0.00684g^2)W^{0.75} \qquad 3.3$$

NEm and NEg are expressed in megacals; W and g are expressed in kilograms.

Equation 3.3 may be reduced to the form illustrated in equation 3.4 to express the daily rate of gain as a function of NEg and W.

$$g = \frac{\sqrt{0.002779 + \frac{0.02736}{W^{0.75}} \text{NEg}} - 0.05272}{0.01368} \qquad 3.4$$

The partial derivative of equation 3.4 with respect to NEg (equation 3.5) indicates that the marginal product of NEg is positive for all levels of NEg.

$$\frac{\partial g}{\partial \text{NEg}} = \left(\frac{1.0}{W^{0.75}}\right) \left(0.002779 + \frac{0.02736}{W^{0.75}} \text{NEg}\right)^{-1/2} > 0 \qquad 3.5$$

The second derivative of equation 3.4 with respect to NEg is negative for all levels of NEg.

$$\frac{\partial^2 g}{\partial \text{NEg}^2} = -\left(\frac{0.01368}{W^{1.5}}\right) \left(0.002779 + \frac{0.02736}{W^{0.75}} \text{NEg}\right)^{-3/2} < 0 \qquad 3.5'$$

The economic implications of equations 3.5 and 3.5' are important. Higher rates of gain can be attained only by increasing the NEg level of a ration. In equation 3.5' the rate of increase in g decreases as NEg increases. Assuming feed ingredient costs are constant, the marginal cost of higher gains increases at an increasing rate. A positive relationship exists between expected rate of gain and cost of gain.

The economic problem becomes one of finding the least cost ration that will provide the profit maximizing daily rate of gain. Problems, however, are encountered in using linear programming models and the net energy system to formulate rations because the total net energy value of any particular ingredient changes with the level of total energy intake (5, p. 79).

Several linear programming models have been developed for ration formulation incorporating the net energy system. A computerized cattle feeding program for replacement and ration formulation is presented by Scott and Broadbent (38). The model combines the following elements of a cattle feedlot program: (1) maximization of returns above variable costs per period of time, (2) assessment of alternative replacements for the feedlot, (3) discounting of future income, and (4) minimum cost ration formulation based on the net energy system. The authors indicate that the model is not as sound from a biological theory standpoint as the Brokken model (discussion follows), it is less expensive to run than the Brokken model, and a number of trial runs with both models result in insignificant differences in the rations when using the usual price ranges, price ratios, and energy requirements. The Scott and Broadbent model formulates a cattle ration for a pre-specified rate of gain. The model does not allow for any uncertainty in prices.

Adams, Septh, and Rohwer (1) modified NEm and NEg values for a least cost ration. Their model combines both energy requirements into one constraint.

Brokken presents a linear programming model to formulate beef rations under conditions of both thermal neutrality and environmental stress (5, 6, 7). The model incorporating environmental stress conditions accounts for variation in the amount of NEg in a ration because of heat and chill factors. One model is designed to find the maximum profit rate of gain with respect to feed costs (7, p. 688). All of Brokken's models are designed strictly for ration formulation. No mechanism for purchasing and selling decisions or feed and livestock price uncertainty is incorporated into any of the models.

Other computerized least cost ration programs are available through cooperative extension services in Nebraska (17), Oklahoma (30, 31), and Utah (20). All extension models include restraints for the N.E. system. The rate of gain is determined after the least cost ration has been formulated in the Nebraska and Oklahoma programs. The Utah program finds a least cost ration for a pre-specified rate of gain provided by the user.

A modified form of Brokken's model assuming thermal neutrality was used to determine the cost of the least cost ration (R_{abcd}) for each feeding alternative (A_{abcd}). Following is a summary of the model used.

The N.E. system requires that a portion of the ration be utilized for body maintenance and the remainder for production or gain (7). In a linear programming model this can be specified in two constraints as shown in equations 3.6 and 3.7.

$$\alpha \sum_t a_{1t} X_t = \text{NEm} \qquad 3.6$$

$$(1-\alpha) \sum_t a_{2t} X_t = NEg \qquad 3.7$$

The proportion of total ration utilized for maintenance and gain are α and $(1-\alpha)$, respectively; a_{1t} and a_{2t} are the amounts of NEm and NEg, respectively, available in one unit of the t-th feed ingredient. a_{1t} and a_{2t} are the amounts of energy in an ingredient that an animal can utilize for maintenance and gain, respectively. NEm and NEg are expressed in megacalories of energy per kilogram of dry matter (42, p. 20). Lofgreen and Garrett state the magnitudes of a_{1t} and a_{2t} are functions of the quality of the t-th ingredient, poorer quality feeds have less energy than higher quality feeds (26, p. 800). The procedures used to determine NEm and NEg are discussed by Lofgreen and Garrett. X_t is the quantity of t-th feed ingredient consumed per day. The right hand sides of both equations are determined from equations 3.2 and 3.3, respectively. The total amount of feed consumed per day is ΣX_t.

The problem utilizing the N.E. system in a linear programming model is that both α and the X_t's are unknown in equations 3.6 and 3.7. Brokken illustrates a solution to the problem by dividing the respective right hand sides by α and $(1-\alpha)$, respectively, as shown in equations 3.8 and 3.9.

$$\sum_t a_{1t} X_t = NEm/\alpha = b_{1u} \qquad 3.8$$

$$\sum_t a_{2t} X_t = NEg/(1-\alpha) = b_{2u} \qquad 3.9$$

The problem now becomes one of determining the level of α. The net energy for production value of an ingredient is always less than the net energy for maintenance value of the ingredient (42, pp. 28-46; 7 , p. 687). Hence, $\Sigma a_{1t}X_t > \Sigma a_{2t}X_t$. From NEm/$\alpha$ > NEg/(1-α), it follows that:

$$\alpha < \frac{NEm}{NEm + NEg} \qquad 3.10$$

A narrower limit on α can be specified as shown in inequalities 3.11.

$$\frac{NEm}{a_{1*}(DM)} \leq \alpha \leq 1 - \frac{NEg}{a_{2*}(DM)} \qquad 3.11$$

DM is the dry matter intake constraint of the animal, a_{1*} and a_{2*} are the highest values of all a_{1t}'s and a_{2t}'s, respectively. Inequality 3.11 implies that the proportion of dry matter intake required for maintenance cannot be smaller than when the ration consists entirely of the ingredient containing the highest NEm, nor greater than when the proportion of dry matter intake required to meet the energy requirements for gain is the lowest (7, p. 688).

A separate constraint vector b_{1u} and b_{2u} is calculated and included in the linear program model for each of several α levels satisfying 3.10 and 3.11.

The linear model used to determine R_{abcd} is:

$$\text{Minimize } R_{abcd} = \sum_t C_t X_t \qquad 3.12.1$$

subject to:

$$\Sigma B_u = 1; \text{ selection index} \qquad 3.12.2$$

$$\sum_t a_{1t}X_t - \sum_u b_{1u} B_u = 0; \text{ NEm requirement} \qquad 3.12.3$$

$$\sum_t a_{2t}X_t - \sum_u b_{2u} B_u = 0; \text{ NEg requirement} \qquad 3.12.4$$

$$\sum_t a_{3t}X_t - c_3Y \geq 0; \text{ total protein requirement} \qquad 3.12.5$$

$$-\sum_{t=1}^{6} a_{4t}X_t + a_{47}X_7 \leq 0; \text{ urea restriction} \qquad 3.12.6$$

$$\sum_t a_{5t}X_t - d_4Y \geq 0; \text{ roughage restriction} \qquad 3.12.7$$

$$-\sum_{t=1}^{6} a_{6t}X_t + Y = 0; \text{ dry matter basis restriction} \qquad 3.12.8$$

$$\sum_{t=1}^{6} a_{7t}X_t - e_7 \Sigma B_u \leq 0; \text{ dry matter maximum restriction} \qquad 3.12.9$$

$$\sum_{t=1}^{6} a_{8t}X_t - f_8 \Sigma B_u \geq 0; \text{ dry matter minimum restriction} \qquad 3.12.10$$

R_{abcd} is the cost per day of the ration that will accomplish the specified rate of gain for a given mean weight at least cost for each A_{abcd}. C_t is the price per unit of the t-th ingredient and X_t is the amount of the t-th feed ingredient included in the least cost ration. In equation 3.12.2 each B_u is a vector for the right hand sides determined by substituting a value of α into equations 3.8 and 3.9. Each α level considered requires one B_u vector. Equation 3.12.2 is an α selection index; it requires that at least one of the B_u vectors be selected. Brokken reports that because of the concavity of the cost function over the feasible range of α no more than two adjacent vectors are selected (7, p. 688). The definition of a_{1t}, a_{2t}, b_{1u}, and b_{2u}

associated with equations 3.6 through 3.9 applies to the terms in equations 3.12.3 and 3.12.4. The latter two constraints insure that the net energy requirements are satisfied. a_{3t} represents the amount of crude (total) protein in the t-th ingredient; c_3 is a coefficient that expresses the total protein requirements as a function of total ration weight (Y). Y is expressed in kilograms of dry matter. Inequality 3.12.6 is used to restrict the amount of urea that is allowed in the least cost ration. a_{4t} $(t < 7)$ is the amount of total protein in feed ingredient t. a_{47} is coefficient in the urea vector used to express the upper limit for the ingredient. Inequation 3.12.7 places a minimum restriction on the roughage content of a ration; d_4 expresses the restriction as a percent of ration weight (Y). Constraint 3.12.8 is used to determine the total dry matter weight of the least cost ration. a_{6t}, a_{7t}, and a_{8t} $(t < 7)$ equals unity for all feed ingredients except urea. Inequations 3.12.9 and 3.12.10 are necessary to express upper and lower restrictions, respectively, on the amount of dry matter allowed in the ration. Coefficients e_7 and f_8 quantify the dry matter restrictions. The values of c_3, d_4, e_7, f_8, and a_{47} are specified in the following subsection.

Brokken's ration formulation model contained equations and inequations similar to 3.12.1 through 3.12.4, 3.12.5, and 3.12.8. Brokken used digestible protein rather than total protein (3.12.5) and fiber rather than roughage (3.12.7). In addition, the Brokken model placed an upper limit on the amount of dry matter intake but not a lower limit (3.12.9 - .10).

2. Data requirements

The selection of feed ingredients and basic ration constraints was established after visiting with personnel from the Animal Science Department, Iowa State University.[1] The nutrient concentration of the selected feed ingredients was determined from Nutrient Requirements of Beef Cattle (42). The ingredients considered and their nutrient concentration are shown in table 3.2.

Animal energy requirements were determined from equations 3.2 and 3.3. Animal nutrient requirements were broadly stated as:

1. Total protein must be greater than or equal to 11 percent of dry matter in the ration (3.12.5); coefficient c_3 equals 0.11.

2. No more than one-third of the protein can come from urea (3.12.6); setting coefficient a_{47} equal to two times the value of a_{37} insures this constraint is satisfied.

3. Ration must contain a minimum of 10 percent dry matter in the form of a roughage (3.12.7); coefficient d_4 equals 0.10.

4. Dry matter must range between 2 and 2.5 percent of body weight; coefficients e_7 and f_8 equal 2 and 2.5 percent of the mean animal weight.

Monthly average prices of alfalfa hay, corn, cottonseed meal, and soybean meal in Iowa were obtained from United States Department of Agriculture publications. Grain sorghum prices were not available for Iowa; therefore, a Nebraska price was used. Prices quoted for grains and alfalfa hay are prices received by farmers.

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[1] Interview with Mitchell R. Geasler, Animal Science Extension Specialist, Iowa State University, Ames, Iowa, June 17, 1975.

Table 3.2. Feed ingredients considered in the ration formulation model and their respective nutrient composition (units per kilogram)

Ingredient t			Nutrient					
Number t	Name	Reference number	NEm[a,b]	NEg[a,b]	Total protein[a,b]	Roughage[b,c]	Dry matter[b]	Dry matter (as fed)[a]
			mcal	mcal	kg	kg	kg	%
1	Alfalfa	1-00-063	1.24	.59	.171	1	1	89.2
2	Corn	4-02-931	2.28	1.48	.100		1	89.0
3	Corn silage	3-08-153	1.56	.99	.081	.5	1	40.0
4	Cottonseed meal	5-01-621	1.69	1.11	.448		1	91.5
5	Grain sorghum	4-04-444	1.85	1.23	.124		1	89.0
6	Soybean meal	5-04-604	1.93	1.29	.515		1	89.0
7	Urea				2.810			

[a]Source: (42).

[b]100 percent dry basis.

[c]Coefficients provided by Mitchell R. Geasler, Animal Science Extension Specialist, Iowa State University.

A ten cents per bushel charge was added to the corn price, forty-two cents per hundredweight to grain sorghum price, and five dollars per ton to the hay price to compensate for elevator, handling, and/or transportation charges to the feedlot.[1] It was assumed price quotes for cottonseed and soybean oil meal include delivery.

Prices for corn silage are not reported, primarily because it is not a feed ingredient that is frequently sold. Corn silage prices were determined as a function of opportunity income per acre of corn grain. It is assumed that a farmer producing corn would have to be compensated an equal amount whether the crop was harvested for grain or silage. If the price of corn is \$1.50 per bushel, then, as shown in table 3.3, the value of silage in corn equivalents is \$12.47 per ton. Since most silage is purchased at harvest, a September corn price was used as the basis for determining silage price.

Urea prices were furnished by an Iowa farm cooperative.

All coefficients used in the linear program to determine least cost rations are expressed in metric units. Ingredient nutrient coefficients were determined on a dry matter basis. It was necessary to convert quoted feed prices to a comparable basis. Equation 3.13 was used to convert all feed ingredient prices except urea.

$$[(QP \div .01\ DM\%) \div Wt] \times 2.2046 = LPP = C_t \quad t = 1, 2, \ldots, 6 \qquad 3.13$$

QP = quoted price

[1]Interview with Dr. Paul Doak, Associate Professor, Economics Department, Iowa State University, and R. G. Hull, Farmers Grain Dealers Association of Iowa, Des Moines, Iowa, June 10, 1975.

Table 3.3. Technique for estimating the value of corn silage in terms of opportunity income for corn grain priced at $1.50 per bushel

Production and harvesting cost differences		Corn grain			Corn silage	
item		amount	dollars		amount	dollars
Estimated yield		110.0 bu.			14.2 tons	
Field loss in harvesting[a]	(6.0%)	6.6 bu.		(1.0%)	0.2 tons	
Harvest yield		103.4 bu.			14.0 tons	
Field losses salvaged by livestock[a]	(0.5%)	2.2 bu.		(0.5%)	0.1 tons	
Realized yield		105.6 bu.			14.1 tons	
Harvest and drying costs		18¢/bu.	18.61		$2.00/ton	[illegible]
Chopping dry cornstalks		$2.00/acre	2.00			
Additional fertilizer					$10.00/acre	10.[illegible]
Total estimated costs			20.61			38.00
Opportunity income of corn grain (105.6 bu. x $1.50)						$158.40
Added costs associated with growing and harvesting silage per acre ($38.00 - $20.61)						17.39
Total opportunity cost of one acre of corn silage (14.1 tons)						$175.79
Required revenue per ton of corn silage to be equivalent to corn grain production ($175.79 ÷ 14.1 tons)						$ 12.47

[a]Source: (18).

DM% = percent dry matter (see table 3.2)

Wt = weight in pounds associated with price quote

LPP = C_t value in linear program, price per kilogram of dry matter of t-th ingredient

Urea is used as a protein source and is assumed 100 percent dry matter; therefore, equation 3.14 was used to convert quoted urea prices to a price per kilogram.

$$QP \times 2.2 = LPP = C_7 \qquad 3.14$$

Feed prices used in the linear programming model for the respective months appear in table A-1.

3. Example of a least cost ration

The cost of least cost ration (R_{abcd}) was determined for each feeding activity (A_{abcd}) described in table 3.1. C_t values were updated at each decision node to reflect existing feed prices. b_{1u} and b_{2u} coefficients were calculated for individual feeding activities over the feasible alpha range. Inequality 3.10 and inequality 3.11 were utilized to determine the feasible alpha range. The b_{1u} and b_{2u} coefficients are a function of expected daily rate of gain (g) and livestock weight (W). The net energy requirements used in determining b_{1u} and b_{2u} were calculated using equations 3.2 and 3.3.

All a_{it} coefficients ($i = 1, \ldots, 8$; $t = 1, \ldots, 7$) used in the linear program except a_{47} appear in table 3.2. Coefficient a_{47} was 5.62 (2 x 2.81). Coefficients c_3 and d_4 were set at the constant level discussed in the previous subsection.

The mean cattle weights for all A_{abcd} (table 3.1) were used to calculate nutrient requirements that are a function of W. Theoretically, these nutrient requirements would change each day, assuming animal weight increases daily.

A linear program tableau for determining R_{4400} is shown in table 3.4. The tableau is for a yearling steer fed at rate of gain four [three pounds (1.36 kg) per day] during the first production period. The mean animal weight during the second decision interval is 968 pounds (439.08 kg) and its daily rate of gain is three pounds (1.36 kg). NEm and NEg requirements are 7.39 and 8.09 megacalories, respectively. The dry matter content of the ration could vary between 8.78 and 10.98 kilograms (3.12.9 and 3.12.10). The lower and upper dry matter limits were specified as a function of two and two and one-half percent of the mean animal weight, respectively.

The feasible alpha range for the problem in table 3.4 was determined to be between .36 and .38 using the lower dry matter constraint and between .29 and .48 when the maximum allowable dry matter intake was used in inequation 3.11. Since the dry matter content of the ration was allowed to vary, an alpha range was used that included the smallest domain for all feasible alpha levels resulting from inequation 3.10 and inequalities 3.11. For R_{4400} the alpha range was .29 to .48. The bottom line in table 3.4 is not a constraint but illustrates the alpha level used in determining the b_{1u} and b_{2u} coefficients. b_{1u} and b_{2u} were determined using the net energy requirements and alpha level as shown in equations 3.8 and 3.9.

Table 3.4. Linear program tableau for determining the daily cost of the least cost ration (R4400) for a 968 pound (439.08 kg) steer gaining 3.00 pounds (1.36 kg) per day

							Activities			
X_1	X_2	X_3	X_4	X_5	X_6	X_7	B_1	B_2	B_3	....
C_1	C_2	C_3	C_4	C_5	C_6	C_7				
							1.0	1.0	1.0	....
1.24	2.28	1.56	1.69	1.85	1.93		-25.48	-24.63	-23.84	....
.59	1.48	.99	1.11	1.23	1.29		-11.39	-11.56	-11.76	....
.171	.100	.81	.448	.124	.515	2.81				
-.171	-.100	-.81	-.448	-.124	-.515	5.62				
1.0		0.5								
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0					
1.0	1.0	1.0	1.0	1.0	1.0		-10.98	-10.98	-10.98	....
1.0	1.0	1.0	1.0	1.0	1.0		-8.78	-8.78	-8.78	
							0.29	0.30	0.31	....

....	B_{19}	B_{20}	W	Constraint Type	Constraint Value	Constraint Name	Constraint
						Objective function	3.12.1
....	1.0	1.0		=	1	Selection imperative	3.12.2
....	-15.72	-15.40		=	0	NEm	3.12.3
....	-15.26	-15.56		=	0	NEg	3.12.4
			-.11	≥	0	Total protein	3.12.5
				≤	0	Urea restriction	3.12.6
			-0.1	≥	0	Roughage	3.12.7
			+1	=	0	Dry matter basis	3.12.8
....	-10.98	-10.98		≤	0	Dry matter maximum	3.12.9
	-8.78	-8.78		≥	0	Dry matter minimum	3.12.10
	0.47	0.48				α level	

The C_t values were determined using equations 3.13 and 3.14 and prices prevailing at each decision node.

In summary, the objective function (3.12.1) changes as feed ingredient prices change. The selection imperative constraint (3.12.2) and constraints 3.12.5 through 3.12.8 were the same for all feeding activities. The net energy constraints (3.12.2 and 3.12.3) were changed as mean weight and expected daily rate of gain varied. The dry matter constraints were changed when mean weight changed.

4. Total feed costs

The linear program model discussed in the previous sections provides the solution to the least cost ration on a per day basis. A daily basis was utilized because nutrient requirements are generally stated as daily requirements. The total feed costs for each feeding activity (F_{abcd}) can be determined by multiplying the daily ration costs (R_{abcd}) by the number of days associated with each feeding period (T_{abcd}). Total feed costs associated with feeding activities during the first through fourth decision intervals were determined by the following equations.

$$F_{aooo} = T_{aooo} R_{aooo} \tag{3.15.1}$$

$$F_{aboo} = T_{aboo} R_{aboo} + F_{aooo} \tag{3.15.2}$$

$$F_{abco} = T_{abco} R_{abco} + F_{aboo} \tag{3.15.3}$$

$$F_{abcd} = T_{abcd} R_{abcd} + F_{abco} \tag{3.15.4}$$

F_{aooo}, F_{aboo}, F_{abco}, and F_{abcd} represent the accumulated feed costs for the respective activities. The R_{abcd} values for 1056 rations used in the economic decision model are shown in table A.2.

F. Procedures for Calculating Costs and Returns of Feeding Activities

The following subsection explains the procedures and assumptions used to calculate variable costs and returns above variable costs for each feeding activity. Variable costs include total feed costs, nonfeed costs such as labor, medicine, and death loss, and purchase price of the feeder animal. Revenue is determined by cattle price levels, weight, and average quality grade of the animals being sold.

1. Total nonfeed variable costs

Total nonfeed variable costs (TNFVC) associated with feeding alternative A are a function of the accumulated number of days the animal was retained in the feedlot and the nonfeed variable costs (NVC) per day per animal plus initial purchase price of the feeder. Equations 3.16.1 through 3.16.4 were used to calculate total nonfeed variable costs for each feeding activity within the four decision intervals.

$$TNFVC_{aooo} = T_{aooo}\ (NVC) + P^{**}_{w}\ (Q_f) \qquad 3.16.1$$

$$TNFVC_{aboo} = T_{aboo}\ (NVC) + TFNVC_{aooo} \qquad 3.16.2$$

$$TNFVC_{abco} = T_{abco}\ (NVC) + TFNVC_{aboo} \qquad 3.16.3$$

$$TNFVC_{abcd} = T_{abcd}\ (NVC) + TFNVC_{abco} \qquad 3.16.4$$

In equation 3.16.1 P^{**}_{w} is the weighted average price paid for feeder cattle and is determined as shown by equation 3.17.

$$P^{**}_{w} = \sum_{j} F^{**}_{j} P^{**}_{j} \qquad 3.17$$

F^{**}_{j} and P^{**}_{j} are defined similar to F^{**}_{ij} and P^{**}_{ij} in equation 3.1. Since all incoming cattle are assumed to weigh 750 pounds, the subscript i is omitted. Incorporating assumption 3 (section D), equation 3.17 can be expanded as shown in 3.17'.

$$P^{**}_{w} = F^{**}_{1} P^{**}_{1} + F^{**}_{2} P^{**}_{2} \quad \text{where} \qquad 3.17'$$

j = 1 = choice and j = 2 = good quality grade cattle.

All TNFVC's include those nonfeed costs associated with the respective feeding activity plus previous feeding activities.

NVC were synthesized from work completed by Trede and Boehlje (44, 4). Costs are based on a 300 head capacity open feedlot with shelter. The referenced publications reported data as cost per hundredweight of gain. The costs were transformed into a per day basis and are shown in table 3.5.

Table 3.5. Nonfeed variable costs per day per animal for feeding yearling steers

Item	Charge
Veterinary and medicine	$ .03
Death loss	.01
Labor	.04
Waste handling	.01
Nonfeed variable costs (NVC)	$ .10

2. Total variable costs

Total variable costs include total feed costs (section E.4) and total nonfeed variable costs (section F.1) for each feeding activity. Equations 3.18.1 through 3.18.4 were used to calculate total variable costs. The variable costs for each feeding activity include that activity plus variable costs of prerequisite feeding activities.

$$VC_{aooo} = TNFVC_{aooo} + F_{aooo} \qquad 3.18.1$$

$$VC_{aboo} = TNFVC_{aboo} + F_{aboo} \qquad 3.18.2$$

$$VC_{abco} = TNFVC_{abco} + F_{abco} \qquad 3.18.3$$

$$VC_{abcd} = TNFVC_{abcd} + F_{abcd} \qquad 3.18.4$$

3. Total revenue

Total revenue received for any pen of cattle is a function of the weighted average price received (P^*_{iw}) and the quantity of beef sold. The quantity of beef available for selling or produced from any feasible feeding activity is indicated by the ending weights in table 3.1. The ending weights are a function of the expected daily rates of gain during each decision interval and the initial weight. P^*_{iw} is determined as shown in equation 3.19.

$$P^*_{iw} = \sum_i \sum_j F^*_{ij} P^*_{ij} \qquad 3.19$$

P^*_{ij} is the price received for i-th weight cattle of j-th quality grade. In this dissertation P^*_{ij} is the price offered for choice (P^*_{i1}) and good (P^*_{i2}) quality grade cattle. F^*_{i1} and F^*_{i2} are the proportion of cattle of i-th weight that grade choice or higher, and good or lower, respectively.

Bullock and Logan (8, 9) and Dinkel and Busch (14, p. 835) reported that quality is a function of animal weight. The latter authors report that quality grade is significantly influenced by an increase in both age (.05 level) and carcass weight (.01 level) of animals slaughtered. Therefore, it is reasonable to assume that as cattle continue through the feed program a proportion will be upgraded.

To determine the relationship between animal weight and quality grades, beef carcass data reports were obtained from Iowa State University cattle feeding experiments conducted at the Allee Experiment Farm, Newell, Iowa. Beef carcass data reports were available for 3,420 steers fed and marketed from 1965 through 1971. Regression analysis was used to quantify the relationship between weight and quality grade. The proportion of cattle grading choice or higher (F^*_{1i}) was used as the dependent variable. The independent variable was liveweight at the time of slaughter.

The frequency distribution of quality grades of cattle slaughtered weighing between 776 and 1,225 pounds was determined according to prespecified weight classes. Classes were divided into 30 pound intervals, i.e., 776 - 805, 806 - 835, ..., 1196 - 1225 pounds, etc. The 30 pound intervals were selected because of the 30 pound intervals associated with consecutive rates of gain considered in feeding activities. One-half pound increments in rate of gain over each decision interval results in 30 pound increments between final weights. The midpoint weight of each interval was selected to represent the class interval.

The midpoint of the first weight interval is 790 pounds; the final weight of feeding activity A_{1000} was 788 pounds.

The proportion of cattle marketed grading choice or higher (F_{i1}) was determined, the remainder graded good or lower. The observed grade distribution by weight class is presented in table 3.6. Because of the weight limits, only 2755 animals were included in the analysis.

Table 3.6. Distribution of quality grades by weight class for yearling slaughter steers weighing between 776 and 1225 pounds

Class	Midpoint	Grade			
		Choice or above (F_{i1})		Good or below (F_{i2})	
no.	lbs.	obs.	%	obs.	%
1	790	6	42.86	8	57.14
2	820	13	41.94	18	58.06
3	850	21	42.86	28	57.14
4	880	47	50.00	47	50.00
5	910	95	66.43	48	33.57
6	940	108	57.14	81	42.86
7	970	183	66.79	91	33.21
8	1000	223	69.25	99	30.75
9	1030	243	77.39	71	22.61
10	1060	269	77.52	78	22.48
11	1090	238	82.35	51	17.65
12	1120	206	82.73	43	17.27
13	1150	164	80.79	39	19.21
14	1180	114	77.55	33	22.45
15	1210	67	74.44	23	25.56

After observing the plot of the dependent variable (F_{i1}) against class midpoints (Wt_i), a quadratic function was used to fit the data. The functional form is shown in equation 3.20.

$$AS\sqrt{pct_i}\sqrt{n_i} = b_o\sqrt{n_i} + b_1\sqrt{n_i}\,Wt_i + b_2\sqrt{n_i}\,Wt_i^2 + e_i \qquad 3.20$$

where $AS\sqrt{pct_i}$ = arcsin transformation of F_{i1}

n_i = number of observations in the i-th weight interval

Wt_i = midpoint in pounds of i-th weight interval.

Weighted regression analysis (41, p. 180) was used because of unequal variances. The unequal variances are attributed to the large variation in the number of observations between weight intervals. The number of observations varied from 6 to 269 per interval. All variables were weighted by the square root of the number of observations (n_i) in the respective weight intervals. An angular or arcsin transformation was made because the dependent data were expressed as percentages or binomial proportions (41, p. 158; 39, p. 327).

Results of regression analysis are shown in table 3.7. The estimated proportion of cattle grading choice or higher (F_{i1}) for each weight interval is shown in table 3.8.

An inconsistency resulted from the $\hat{F}_{i1}$ and what one would expect at weights above 1165 pounds (table 3.8). The estimated proportion of cattle grading choice or higher decreases as animal weights exceed 1165 pounds when the quadratic function is used to estimate F_{i1}. It is not reasonable that the proportion of cattle grading choice or above would decrease as animals are continued on feed once the choice quality grade has been attained.

A possible explanation for the inconsistency in the last three weight intervals is that a large proportion of cattle marketed in these intervals were American beef and possibly mixed dairy breeds of cattle. They were marketed at a state of physiological maturity in which

Table 3.7. Regression analysis coefficients used to determine the relationship between quality grade and live animal weight for yearling steers weighing between 776 and 1225 pounds

	Equational form		
Coefficients	$F_{i1} = AS\sqrt{pct_i} = b_o\sqrt{n_i}$	$+\ b_1\sqrt{n_i}\ Wt_i$	$+\ b_2\sqrt{n_i}\ Wt_i^2$
B values	-272.131	-582	-.0003
T for H_o: B = 0	-3.472**	3.840**	3.445**
Standard error of B	78.370	.152	.00007
R-square	.865		

**Significant at the .01 level.

Table 3.8. Proportion of cattle grading choice or higher by weight group, in the observed data, predicted from regression analysis, and used in the economic model

Class	Midpoint	Observed from data F_{i1}	Predicted from equation $\hat{F}_{i1}$	Used in the economic model F_{i1}	F_{i2}
no.	lbs.	%	%	%	%
1	790	42.86	25.86	25.86	74.14
2	820	41.94	34.36	34.36	65.64
3	850	42.86	42.58	42.58	57.42
4	880	50.00	50.26	50.26	49.74
5	910	66.43	57.13	57.13	42.87
6	940	57.14	63.11	63.11	36.89
7	970	66.79	68.16	68.16	31.84
8	1000	69.25	72.31	72.31	27.69
9	1030	77.39	75.57	75.57	75.57
10	1060	75.52	78.08	78.08	21.92
11	1090	82.35	79.83	79.83	20.17
12	1120	82.73	80.92	80.92	19.08
13	1150	80.79	81.37	81.37	18.63
14	1180	77.55	81.26	81.37	18.63
15	1210	74.44	80.41	81.37	18.63

insufficient intramuscular fat was deposited to reach choice quality grade.[1] The majority of other cattle marketed was assumed to be of a mixed European breed. The latter breeds typically grade choice at lighter weights.

For use in this dissertation, the quadratic function was truncated at 1165 pounds. Animals weighing more than 1165 pounds were assumed to grade 81.37 percent choice.

The distribution of quality grades was used to compute P^*_{iw}. P^*_{iw} for A_{4400} is calculated as shown in equation 3.19'.

[1]Interview with Dr. David G. Topel, Animal Science Department, Iowa State University, Ames, Iowa, July 14, 1975.

$$P^*_{1058,w} = 0.7808\ P^*_{1058,1} + 0.2192\ P^*_{1058,2} \qquad 3.19'$$

Both P^{**}_{ij} (equation 3.19) and P^*_j (equation 3.17) were determined as monthly average prices that prevailed during the month under consideration. All livestock prices were obtained from the United States Department of Agriculture, Consumer and Marketing Service, Livestock Division, Des Moines, Iowa. Slaughter cattle prices used were based on Iowa-Southern Minnesota direct cattle auction market quotations. Feeder cattle prices used were based on Iowa auction market quotations.

USDA quotes monthly average prices for choice and good feeder steers in weight intervals of between 700 - 800 pounds and 800 - 1000 pounds. The weight intervals used for quoting choice and good slaughter cattle prices are 900 - 1100 pounds and 1100 - 1300 pounds. Prices were quoted for other weight intervals for both feeder and slaughter cattle; however, these were not needed in the dissertation. A 100 pound overlap in price quotes exists between heavy feeder cattle and light slaughter cattle; therefore, any animal weighing 950 pounds or less at the end of a feeding activity was considered a feeder animal.

If a monthly average price was not available or quoted for a specific weight interval, it was necessary to estimate the missing value. The estimate was made by extrapolating from prices quoted for other weight intervals during the same month. For example, in August 1972, prices quoted for good 600 - 700 pound and 700 - 800 pound steers were \$38.55 and \$36.53, respectively; no price was available for good steers weighing 800 - 1000 pounds. The extrapolation procedure used is given in equation 3.21.

$$\frac{(P_{7-800})^2}{(P_{6-700})} = \hat{P}_{8-1000} \qquad 3.21$$

$$\frac{(\$36.53)^2}{\$38.55} = \$34.62 \qquad 3.21'$$

The estimated price for good grade feeder cattle weighing between 800 - 1000 pounds during August 1972 at Iowa auction markets is \$34.62 per hundredweight. The estimated prices were used when necessary to determine P^*_{iw} in equation 3.19.

Total revenue realizable for each feeding-marketing activity (TR_{aooos}, TR_{aboos}, TR_{abcos}, TR_{abcds}) was determined by use of equations 3.22.1 through 3.22.4.

$$TR_{aooos} = 0.96\ W_i\ P^*_{iw} \qquad 3.22.1$$

$$TR_{aboos} = 0.96\ W_i\ P^*_{iw} \qquad 3.22.2$$

$$TR_{abcos} = 0.96\ W_i\ P^*_{iw} \qquad 3.22.3$$

$$TR_{abcds} = 0.96\ W_i\ P^*_{iw} \qquad 3.22.4$$

The subscripts a, b, c, and d associated with TR are defined the same as the subscript notation used to describe the set of feasible feeding activities A. The subscript s indicates a selling alternative or action subsequent to the indicated feeding action. The constant 0.96 was used to incorporate the four percent outshrink (assumption 2). W_i is the ending liveweight associated with the respective feeding activity (table 3.1). P^*_{iw} was determined by use of equations 3.19 and 3.20.

4. Returns above variable costs

Returns above variable costs (π) were obtained for each of the feasible feeding activities as shown in equations 3.23.1 through 3.23.4.

$$\pi_{aooo} = TR_{aooos} - VC_{aooo} \qquad 3.23.1$$

$$\pi_{aboo} = TR_{aboos} - VC_{aboo} \qquad 3.23.2$$

$$\pi_{abco} = TR_{abcos} - VC_{abco} \qquad 3.23.3$$

$$\pi_{abcd} = TR_{abcds} - VC_{abcd} \qquad 3.23.4$$

When a comparison of incomes was made for a sequence of feeding and marketing actions, it was necessary to use discounting procedures. The discounting procedures incorporate the opportunity cost of having capital invested in the cattle feeding enterprise when it is possible to invest elsewhere and earn a return or use the invested capital to pay existing debts, if any. The series of equations in 3.24 were used to determine the present value of future incomes for each subsequent decision period remaining in the planning horizon.

$$G_{aooo} = \frac{\pi_{aooo}}{(1+rf)} \qquad 3.24.1$$

at decision node number:

1, f = 1/6

$$G_{aboo} = \frac{\pi_{aboo}}{(1+rf)} \qquad 3.24.2$$

at decision node number:

1, f = 1/3

2, f = 1/6

$$G_{abco} = \frac{\pi_{abco}}{(1+rf)} \qquad 3.24.3$$

at decision node number:

1, f = 1/2

2, f = 1/3

3, f = 1/6

$$G_{abcd} = \frac{\pi_{abcd}}{(1+rf)} \qquad 3.24.4$$

at decision node number:

1, f = 2/3

2, f = 1/2

3, f = 1/3

4, f = 1/6

where

π = return above variable costs for the subscripted feeding activity

r = simple annual interest rate

f = fraction of a year elapsing before π can be realized

G = discounted returns above variable costs for the subscripted feeding activity.

The interest rate (r) used was the average rate charged by the Federal Intermediate Credit Bank during 1972 and 1973. The rates were 6.00 and 7.16 percent for the respective years (46, p. S-18). The letter G was selected to represent the gain or payoff of each action to be consistent with the notation to represent gains in the previously discussed Bayesian decision theory.

G. Bayesian Strategies

In the following subsection the components of a Bayesian decision as discussed in chapter two are defined and quantified in terms used in this dissertation. The procedures used to compute the no data and data Bayesian strategies are presented using the quantified components.

1. Components of the Bayesian decision model

The set of actions A available to the cattle feeder range from not feeding to feeding for various rates of gain and varying the length of time cattle are retained in the feedlot. By assumptions listed in section D, if cattle are fed, the length of time an animal can be retained on feed ranges between 60 and 240 days, and the daily rate of gain varies between 1.5 and 3.0 pounds.

Each A_{aooo}, A_{aboo}, A_{abco}, and A_{abcd} presented in table 3.1 represents a unique combination of one or more rates of gain and variation in length of time on feed. These activities are used to represent the potential cattle feeding and marketing actions available to the cattle feeder.

States of the world (θ) confronting the cattle feeder are changes in the cattle prices that may occur between a decision node and the time they are marketed or intended to be marketed. The change in cattle prices is a random variable over which the cattle feeder has no control. Prices may increase, remain constant, or decrease from their present level. The magnitude of states of the world were determined by use of equation 3.25.

$$\Delta P_{ct+i} = P_{ct+i} - P_{ct} \qquad \text{where} \qquad 3.25$$

ΔP_{ct+i} = change in average price of choice slaughter cattle i (i = 2, 4, 6, 8) months after decision node t (t = 1, 2, 3, 4)

P_{ct+i} = price i months after decision node t

if $t = 1$, then i = 2, 4, 6, 8

if $t = 2$, then i = 2, 4, 6

if $t = 3$, then i = 2, 4

if $t = 4$, then $i = 2$

P_{ct} = average price of choice slaughter cattle in the base period at t-th decision node (t = 1, 2, 3, 4).

Changes in cattle price levels were calculated for two, four, six, and eight month intervals. The eight month interval or time period coincides with the length of the planning horizon. The magnitude and frequency of ΔP_{ct+i}'s were determined using Interior Iowa-Southern Minnesota monthly average choice slaughter cattle prices. The average price P_{ct} was determined by calculating the mean price for 900 - 1100 pound and 1100 - 1300 pound choice slaughter steers.

For the period extending from June 1965 through 1971, there were 77, 75, 73, and 71 ΔP_{ct+i}'s calculated for intervals of two, four, six, and eight months in length, respectively. The ΔP_{ct+i}'s calculated represent 72, 70, 70, and 69 states of the world for the respective time periods. To reduce the number of θ's being considered in each consecutive two month time period, classes representing ranges of ΔP_{ct+i}'s were identified. The midpoint of each class was used to represent a specific state of the world. The states of the world considered for each time period are shown in table 3.9.

Table 3.9. States of the world and values of θ used in the Bayesian decision model to assist in making cattle feeding and marketing decisions for two, four, six, and eight months in the future

	Elapse time (months)			
	i = 2		i = 4	
State (θ_{ij})	Range in value of ΔP_{ct+2}	Value of θ_{1j}	Range in value of ΔP_{ct+4}	Value of θ_{2j}
		dollars per hundredweight		
θ_{i1}	$3.51 \le \Delta P_{ct+2}$	5.00	$3.86 \le \Delta P_{ct+4}$	5.50
θ_{i2}	$1.51 \le \Delta P_{ct+2} \le 3.50$	2.50	$1.66 \le \Delta P_{ct+4} \le 3.85$	2.75
θ_{i3}	$0.51 \le \Delta P_{ct+2} \le 1.50$	1.00	$0.56 \le \Delta P_{ct+4} \le 1.65$	1.10
θ_{i4}	$-0.50 \le \Delta P_{ct+2} \le 0.50$	0.0	$-0.55 \le \Delta P_{ct+4} \le 0.55$	0.0
θ_{i5}	$-1.50 \le \Delta P_{ct+2} \le -0.51$	-1.00	$-1.65 \le \Delta P_{ct+4} \le 0.56$	-1.10
θ_{i6}	$\Delta P_{ct+2} \le -1.51$	-2.50	$-3.85 \le \Delta P_{ct+4} \le -1.66$	-2.75
θ_{i7}			$\Delta P_{ct+4} \le -3.86$	-5.50

i = 6		i = 8	
Range in value of ΔP_{ct+6}	Value of θ_{3j}	Range in value of ΔP_{ct+8}	Value of θ_{4j}
$4.21 \leq \Delta P_{ct+6}$	6.00	$4.56 \leq \Delta P_{ct+8}$	6.50
$1.81 \leq \Delta P_{ct+6} \leq 4.20$	3.00	$1.96 \leq \Delta P_{ct+8} \leq 4.55$	3.25
$0.61 \leq \Delta P_{ct+6} \leq 1.80$	1.20	$0.66 \leq \Delta P_{ct+8} \leq 1.95$	1.30
$-0.60 \leq \Delta P_{ct+6} \leq 0.60$	0.0	$-0.65 \leq \Delta P_{ct+8} \leq 0.65$	0.0
$-1.80 \leq \Delta P_{ct+6} \leq -0.61$	-1.20	$-1.95 \leq \Delta P_{ct+8} \leq -0.66$	-1.30
$-4.20 \leq \Delta P_{ct+6} \leq -1.81$	-3.00	$\Delta P_{ct+8} \leq -1.96$	-3.25
$\Delta P_{ct+6} \leq -4.21$	-6.00		

θ_{ij} represents the j-th state of the world that can occur i months (i = 2, 4, 6, 8) after decision node t (t = 1, 2, 3, 4). When i equals 2 or 8, j equals 1, 2, ..., 6; when i equals 4 or 6, j equals 1, 2, ..., 7. The respectives i's represent time periods two, four, six, or eight months into the future. Variation existed in the range of θ_j between the i periods. The absolute level and range of ΔP_{t+i}'s generally increased as i increased. From June 1965 through 1971, the ranges of price changes for two, four, six, and eight month intervals were \$-4.00 to \$5.37, \$-6.06 to \$6.06, \$-5.66 to \$5.82, and \$-4.88 to \$7.14, respectively. Because of the trend of increase in level of price changes with respect to time, each θ_{ij} considered in subsequent periods contained a larger interval. States of the world θ_{i1} (for all i's), θ_{i6} (for i = 2, 8), and θ_{i7} (for i = 4, 6) are open ended to allow for price changes larger in absolute value. Reasons for including six rather than seven classes for four and six month periods will be discussed later in the dissertation.

The consequences [C(A,θ)] for each combination of an action and state of the world were computed by adjusting P^*_{iw} (equation 3.19) for θ_{ij} and then recomputing gains for each activity as previously explained (3.22, 3.23, and 3.24). Following is an explanation of the procedures used to adjust P^*_{iw}.

Recall that the states of the world θ represent changes in the average price of choice slaughter cattle. When choice slaughter cattle prices change, it can reasonably be expected that the prices of other grades and weight groups of cattle will change (45, p. 8; 9, p. 15). Hence, the gains for all A_{aooo}, A_{aboo}, A_{abco}, and A_{abcd} need to be determined as a function of the prevailing state of the world θ_{ij} and average price of choice slaughter cattle at decision node t (P_{ct}).

The initial step was to determine the relationship between prices of various grades and weight groups of cattle and P_{ct} at decision node t. To reduce the complexity of the procedures, it was assumed that the same price relationship would exist in time period i as currently exists. The price relationships were determined by the series of equations 3.26.1 through 3.26.8.

$$A_t = .5 \sum_{i=t-1}^{i=t} (P_{ci11-13} - P_{ci}) \qquad 3.26.1$$

$$B_t = .5 \sum_{i=t-1}^{i=t} (P_{ci9-11} - P_{ci}) \qquad 3.26.2$$

$$C_t = .5 \sum_{i=t-1}^{i=t} (P_{ci8-10} - P_{ci9-11}) \qquad 3.26.3$$

$$D_t = .5 \sum_{i=t-1}^{i=t} (P_{ci7-8} - P_{ci9-11}) \qquad 3.26.4$$

$$E_t = .5 \sum_{i=t-1}^{i=t} (P_{gi11-13} - P_{ci9-11}) \qquad 3.26.5$$

$$F_t = .5 \sum_{i=t-1}^{i=t} (P_{gi9-11} - P_{ci9-11}) \qquad 3.26.6$$

$$G_t = .5 \sum_{i=t-1}^{i=t} (P_{gi8-10} - P_{ci9-11}) \qquad 3.26.7$$

$$H_t = .5 \sum_{i=t-1}^{i=t} (P_{gi7-8} - P_{ci9-11}) \qquad 3.26.8$$

P_{ci} = average price per hundredweight for choice slaughter steers t-i months before decision node t

$P_{ci11-13}$ = average price per hundredweight for 1100 - 1300 pound choice steers t-i months before decision node t

P_{ci9-11} = average price per hundredweight for 900 - 1100 pound choice steers t-i months before decision node t

P_{ci8-10} = 800 - 1000 pound choice steers t-i months before decision node t

P_{ci7-8} = average price per hundredweight for 700-800 pound choice steers t-i months before decision node t

$P_{gi11-13}$ = average price per hundredweight for 1100 - 1300 pound good steers t-i months before decision node t

P_{gi9-11} = average price per hundredweight for 900 - 1100 pound good steers t-i months before decision node t

P_{gi8-10} = average price per hundredweight for 800 - 1000 pound good steers t-i months before decision node t

P_{gi7-8} = average price per hundredweight for 700 - 800 pound good steers t-i months before decision node t

A_t, ..., H_t = the average deviation between average prices for specified weight groups and/or quality grades of steers for the previous two months and average choice slaughter steer prices or prices of choice slaughter steers weighing 900 - 1100 pounds at decision node t.

Equations 3.26.1 through 3.26.8 determine the average price differentials for all weight groups and quality grades of steers required in the dissertation as a function of their respective prices for the previous two months and the average price of all choice slaughter steers or choice steers weighing between 900 - 1100 pounds during the same period.

To utilize these price differentials and extrapolate price relationships into time period i as a function of θ_{ij}, the following series of equations were required.

$$\hat{P}_{ct+i11-13} = A_t + (P_{ct} + \theta_{ij}) \qquad 3.27.1$$

$$\hat{P}_{ct+i9-11} = B_t + (P_{ct} + \theta_{ij}) \qquad 3.27.2$$

$$\hat{P}_{ct+i8-10} = C_t + (P_{ct9-11} + \theta_{ij}) \qquad 3.27.3$$

$$\hat{P}_{ct+i7-8} = D_t + (P_{ct9-11} + \theta_{ij}) \qquad 3.27.4$$

$$\hat{P}_{gt+i11-13} = E_t + (P_{ct9-11} + \theta_{ij}) \qquad 3.27.5$$

$$\hat{P}_{gt+i9-11} = F_t + (P_{ct9-11} + \theta_{ij}) \qquad 3.27.6$$

$$\hat{P}_{gt+i8-10} = G_t + (P_{ct9-11} + \theta_{ij}) \qquad 3.27.7$$

$$\hat{P}_{gt+i7-8} = H_t + (P_{ct9-11} + \theta_{ij}) \quad \text{where} \qquad 3.27.8$$

θ_{ij} = j-th state of the world that can occur in the i-th time period (i = 2, 4, 6, 8), (if i = 2 or 8, j = 1, ..., 6; if i = 4 or 6, j = 1, ..., 7)

$\hat{P}_{jt+iw}$ = expected price for w-th weight, j-th quality grade of cattle (c = choice, g = good) at t-th decision node i months in the future.

Equations 3.27.1 through 3.27.8 are the model used to determine $\hat{P}_{jt+iw}$. An equation similar to 3.19 was used to determine the weighted price for cattle at decision node t+i.

$$\hat{P}^*_{t+iw} = \sum_j \sum_w F^*_{jw} \hat{P}_{jt+iw} \qquad \text{where} \tag{3.19'}$$

$\hat{P}^*_{t+iw}$ = expected weighted price for cattle of weight group w, i months in the future from decision node t

F^*_{jw} = proportion of cattle of j quality grade in w-th weight group

$\hat{P}_{jt+iw}$ = expected price for w-th weight, j-th quality grade of cattle i months in the future from decision node t

$\hat{P}^*_{t+iw}$ was then used in equation 3.22' to determine the total revenue given the state of the world (TR_{ij}). Equations 3.23' and 3.24' were used to calculate the gains for each specified combination action-state of the world [C(A,θ)] as shown below.

$$TR_{ij} = 0.96\ w\ \hat{P}^*_{t+iw} \tag{3.22'}$$

$$\pi_{ij} = TR_{ij} - VC \tag{3.23'}$$

$$G_{ij} = \frac{\pi_{ij}}{(1+rf)} \tag{3.24'}$$

TR, π, and G are conditional on the prevailing state of the world θ_j in time period i.

Table 3.10 shows the gains from 3.24' for all combinations of A_{aooo}, θ_{2j}. These gains were used in determining the Bayesian strategies at the

Table 3.10. Gains per head for five cattle feeding-marketing actions and six states of the world for a two month production period, December 1971

	State of the world θ_{2j}					
Action	θ_{21}	θ_{22}	θ_{23}	θ_{24}	θ_{25}	θ_{26}
			dollars			
A_{0000}	0.00	0.00	0.00	0.00	0.00	0.00
A_{1000}	26.27	7.53	-3.69	-11.19	-18.67	-29.91
A_{2000}	17.83	-1.61	-13.28	-21.05	-28.82	-40.49
A_{3000}	27.65	7.51	-4.58	-12.64	-20.70	-32.79
A_{4000}	36.43	15.65	3.04	-5.31	-13.64	-26.17

initial decision node in the sample period (December 1971) for the first production period.

The initial prior probability $P(\theta_{ij})$ for the i-th time period and j-th state of the world was determined by calculating the relative frequency that each state of the world had occurred between June 1965 and December 1971. The initial frequencies and prior probabilities for all θ_{ij} are shown in table 3.11. These priors were used in determining the initial Bayesian no data strategies. For subsequent no data decisions, $P(\theta_{ij})$ was updated to include the most recent observations. For example, the monthly average choice slaughter steer prices from November 1971 through February 1972 were \$33.14, \$34.04, \$35.17, and \$35.90 per hundredweight, respectively. The two month price increases between November and January, and December and February, were \$2.03 and \$1.86, respectively. Both values are contained in the

Table 3.11. Frequency of occurrence and prior probability of price changes from June 1965 through 1971

State of the world	ΔP_{ct+2} Frequency of θ_{2j}	ΔP_{ct+2} $P(\theta_{2j})$	ΔP_{ct+4} Frequency of θ_{4j}	ΔP_{ct+4} $P(\theta_{4j})$	ΔP_{ct+6} Frequency of θ_{6j}	ΔP_{ct+6} $P(\theta_{6j})$	ΔP_{ct+8} Frequency of θ_{8j}	ΔP_{ct+8} $P(\theta_{8j})$
θ_{i1}	3	0.039	5	0.067	4	0.055	4	0.056
θ_{i2}	12	0.156	11	0.146	16	0.219	15	0.211
θ_{i3}	18	0.234	18	0.240	17	0.233	19	0.268
θ_{i4}	19	0.247	19	0.253	17	0.233	17	0.239
θ_{i5}	14	0.181	12	0.160	8	0.110	8	0.113
θ_{i6}	11	0.143	8	0.107	9	0.123	8	0.113
θ_{i7}	n.a.[a]	n.a.	2	0.027	2	0.027	n.a.	n.a.
Totals	77	1.000	75	1.000	73	1.000	71	1.000

[a]Not applicable.

range specified by θ_{22}. At decision node one, December 1971, the observed frequency of θ_{22} was 12, at decision node two, February 1972, the observed frequency of θ_{22} was 14.

The $P(\theta)$ is the only probability distribution required when determining the no data Bayesian strategy. To incorporate sample information into the data Bayesian strategy, one additional probability component is required. Information concerning the historical accuracy of the sample information or forecast source is utilized. This information, $P(Z_{ik}|\theta_{ij})$, is the likelihood of forecast Z_k (k = 1, ..., 7) being made when θ_j is the true state of the world in the i-th time period (i = 2, 4, 6, 8 months) in the future. $P(Z_{ik}|\theta_{ij})$ was determined by comparing all cattle price forecasts that appeared in the Iowa Farm Outlook letter between June 1965 and December 1971 with actual prices that occurred for two, four, six, and eight months after the respective forecasts were made and then computing percentages. $P(Z_{ik}|\theta_{ij})$ is the conditional probability of observing forecast Z_k (k = 1, ..., 7) given that θ_j is the true state of the world i months in the future.

$P(Z_{2k}|\theta_{2j})$, (k = 1, ..., 7; j = 1, ..., 6), based on two month slaughter cattle forecasts appearing in the Iowa Farm Outlook letter from June 1965 through December 1971 are shown in table 3.12. $P(Z_{ik}|\theta_{ij})$ was updated prior to each subsequent decision after December 1971 to reflect results of more recent observations appearing in the Outlook letter. Procedures similar to those described for updating the priors were used to update the likelihoods. If the letter had contained "perfect market information," the diagonal elements (k = j) would be

Table 3.12. $P(Z_{2k}|\theta_{2j})$ for cattle price forecasts appearing in the Iowa Farm Outlook letter from June 1965 through 1971

	$P(Z_{2k}\|\theta_{2j})$						
	Z_{2k}						
θ_{2j}	Z_{21}	Z_{22}	Z_{23}	Z_{24}	Z_{25}	Z_{26}	Z_{27}
θ_{21}	0.000	0.000	1.000	0.000	0.000	0.000	0.000
θ_{22}	0.000	0.000	0.250	0.500	0.000	0.250	0.000
θ_{23}	0.000	0.091	0.091	0.364	0.455	0.000	0.000
θ_{24}	0.000	0.000	0.167	0.444	0.222	0.167	0.000
θ_{25}	0.000	0.000	0.000	0.500	0.167	0.333	0.000
θ_{26}	0.000	0.000	0.000	0.333	0.333	0.167	0.167

1.000 percent for four and six month forecasts. For two and eight month forecasts, the sum of $P(Z_{i6}|\theta_{i6})$ and $P(Z_{i7}|\theta_{i7})$ where i equals 2 and 8, respectively, would equal 1.000 if the letter contained perfect information. The class intervals for θ_{26} and θ_{86} are open ended (table 3.9); the upper boundary for θ_{26} and θ_{86} is equal to the upper boundaries for Z_{26} and Z_{86}, respectively.

Seven states of the world θ_j ($j = 1, \ldots, 7$) were utilized in determining the no data and data Bayesian strategies for four and six month periods and only six states of the world θ_j ($j = 1, \ldots, 6$) for periods of two and eight months into the future. The reason for fewer states of the world in two and eight month periods is that θ_{27} and θ_{87} never were observed following all Z_{2k} and Z_{8k} forecasts between 1965 and 1971. The conditional probabilities expressed by equations 3.28.1 and 3.28.2 were zero.

$$P(Z_{2k}|\theta_{27}) = 0 \quad \text{for } k = 1, \ldots, 7 \qquad 3.28.1$$

$$P(Z_{8k}|\theta_{87}) = 0 \quad \text{for } k = 1, \ldots, 7 \qquad 3.28.2$$

The sum of the conditional probabilities given θ_{27} and θ_{87} do not sum to one.

$$\sum_{k=1}^{k=7} P(Z_{2k}|\theta_{27}) = 0.0 \qquad 3.29.1$$

$$\sum_{k=1}^{k=7} P(Z_{8k}|\theta_{87}) = 0.0 \qquad 3.29.2$$

Equations 3.29.1 and 3.29.2 demonstrate the inconsistency previously discussed (equation 2.2.2). θ_{26} and θ_{86} were open ended intervals to include all price decreases lower than \$1.51 and \$1.96 for two and eight month periods, respectively.

Five problems were encountered in calculating $P(Z_{ik}|\theta_{ij})$. Until the fall of 1968, price forecasts appearing in the Outlook letter were based on the Chicago livestock market. Since the fall of 1968 all price forecasts are based on Iowa markets. This change required the utilization of additional market data (Chicago prices) to determine the ΔP_{ct+i}'s.

The second problem was that price forecasts frequently covered a range of values. Statements such as "I expect choice steer prices in Iowa to be within the \$30 - \$32 range during the May - August period" appeared frequently (24b). This type of statement is not used solely by the current editor of the Iowa Farm Outlook letter; it is a technique also used by other forecasters (23, p. 12). It is assumed that the forecast covers the range of prices expected during the referenced period.

The problem encountered with a range of forecast prices was in selecting a specific price from within the forecast range to use in computing expected gains or payoffs. The assumed distribution for occurrence of all prices within the forecast range can influence the specific price selected. It was assumed that the probability of occurrence of various prices within the forecast range was symmetric and unimodal. Therefore, it is reasonable to select the mean of the forecast range as a representative price. The mean forecast price was compared with the true state of the world.

The predicted price change Z_{ik} was calculated as shown in equation 3.30.

$$Z_{ik} = \hat{P}_{ct+i} - P_{ct} \quad \text{where} \qquad 3.30$$

Z_{ik} = predicted price change k (k = 1, ..., 7) for the i-th time period (i = 2, 4, 6, 8) in the future

$\hat{P}_{ct+i}$ = mid-range of forecast prices for choice slaughter cattle appearing in the Iowa Farm Outlook letter at decision node t for i months in the future (i = 2, 4, 6, 8)

P_{ct} = most recent average market prices quoted in the Iowa Farm Outlook letter for choice slaughter steers at decision node t.

The actual price change θ_{ij} was calculated as shown in equation 3.31.

$$\theta_{ij} = P_{ct+i} - P_{ct} \quad \text{where} \qquad 3.31$$

P_{ct+i} = the observed average price for choice slaughter cattle i months after decision node t (i = 2, 4, 6, 8).

The third problem encountered in utilizing price forecast information appearing in the Iowa Farm Outlook letter was interpreting qualitative terms used to forecast prices. Two questions arise concerning the interpretation of qualitative terms; first, what message did the forecaster really want to convey, and secondly, how will the qualitative terms be interpreted by the decision maker. Is the decision maker's definition or interpretation consistent with the price forecaster's intended implication? A certain amount of subjectivity is involved by the forecaster and the decision maker in quantifying qualitative statements. To be consistent when interpreting qualitative statements appearing in successive forecasts, a constant value or definition was assigned to each statement at its initial appearance. Qualitative statements frequently appearing in the Outlook letter and the quantitative definition assessed each term by the author are shown in table 3.13. The qualitative terms include both prices and seasonal or time aspects of market forecasts.

An example of the fourth problem encountered in computing $P(Z_{ik}|\theta_{ij})$ can be observed by referring to the quotation previously included in the discussion of problem two. Frequently price forecasts were made for periods of time encompassing several months, i.e., "... May - August period." A forecast involving more than one month generates a problem in determining the observed state of the world θ_{ij} for each month. The observed state of the world was necessary in calculating $P(Z_{ik}|\theta_{ij})$. At no time during the study period (June 1965 - December 1973) was P_{ct} the same for two consecutive months.

Table 3.13. Qualitative statements appearing in the Iowa Farm Outlook letter and assessed interpretations

Statement	Interpretation
Expected to remain strong	Term used following price rises, assumed no further price changes
Moderate price increase	$3.00 - 4.00 per cwt. increase from current levels
Upward pressures	$1.00 - 4.00 per cwt. increase from current levels
Some strength	$1.00 - 2.00 per cwt. increase from current levels
Near or a little above	No change to $1.00 increase per cwt. from current levels
Somewhat higher Slight strength May strengthen	$.50 - 1.00 per cwt. increase from current levels
Fairly steady Fairly close	No change from current levels
May weaken Slight weakness Somewhat lower	$.50 - 1.00 per cwt. decrease
Near or a little below	No change to $1.00 decrease from current levels
Some weakness	$1.00 - 2.00 decrease from current levels
Downward pressures	$1.00 - 4.00 decrease from current levels
Moderate price decrease	$3.00 - 4.00 decrease from current levels
Mid to upper $30's	$35.00 - 38.00 range
Mid to high $30's	$35.00 - 39.00 range

Table 3.13. Continued

Statement	Interpretation
Low $30's	$31.00 - 33.00 range
Fall	September, October, and November
Winter	December, January, and February
Spring	March, April, and May
Summer	June, July, and August
First quarter	January, February, and March
Second quarter	April, May, and June
Third quarter	July, August, and September
Fourth (last) quarter	October, November, and December

To determine θ_{ij} (equation 3.31), P_{ct+i} was set equal to the mean price that occurred during the forecast period. If the forecast extended beyond one decision period (i), then Z_k and θ_j were included as an observation for computing $P(Z_{ik}|\theta_{ij})$ for all i periods included in the forecast range.

The fifth problem encountered was that the required forecasts for i months in the future were not always available at each decision node t. When

t = 1; required forecasts were i = 2, 4, 6, 8

t = 2; required forecasts were i = 2, 4, 6

t = 3; required forecasts were i = 2, 4

t = 4; required forecasts were i = 2

If the i-th forecast were not available at decision node t, then the most recent i-th forecast made within the last 60 days was utilized. If the i-th forecast did not appear in the Outlook letter within the past 60 days, the next closest forecast (i - 2) was used. For example, if at decision node one, December 1971, an eight month forecast ($\hat{P}_{ct+8}$) was not available, then the most recent $\hat{P}_{ct+8}$ made in November or October was substituted. If no $\hat{P}_{ct+8}$'s were made within the last 60 days, $\hat{P}_{ct+6}$ was used in place of $\hat{P}_{ct+8}$.

The posterior probability $P(\theta_{ij}|Z_{ik})$, i.e., the conditional probability of j-th state of the world occurring given the k-th prediction for the i-th time period, was calculated by determining the joint probabilities for all θ_j and Z_k in each of the i time periods and normalizing. A variation of equation 2.1 was used to determine all posterior probabilities for each respective time period (i).

$$P(\theta_{ij}|Z_{ik}) = \frac{P(\theta_{ij})\ P(Z_{ik}|\theta_{ij})}{P(Z_{ik})} \qquad \text{where} \qquad 2.1*$$

$$P(Z_{ik}) = \sum_j P(\theta_{ij})\ P(Z_{ik}|\theta_{ij}) \qquad 2.1'*$$

A unique set of posterior probabilities was determined for each i time period, i.e., two, four, six, or eight month forecasts. Table 3.14 shows the posterior distribution of price change used in determining the initial Bayesian data strategies for a two month period (i = 2). The posterior probabilities were updated prior to making all subsequent decisions. Recomputing $P(\theta_{ij}|Z_{ik})$ at each decision point provided for the inclusion of most recent market information via the continuously changing priors $P(\theta_{ij})$ and sample information $P(Z_{ik}|\theta_{ij})$.

Table 3.14. Posterior distribution $[P(\theta_{2j}|Z_{2k})]$ for cattle price forecasts appearing in the Iowa Farm Outlook letter from June 1965 through 1971

	$P(\theta_{2j}\|Z_{2k})$						
	Z_{2k}						
θ_{2j}	Z_{21}	Z_{22}	Z_{23}	Z_{24}	Z_{25}	Z_{26}	Z_{27}
θ_{21}	0.000	0.000	0.278	0.000	0.000	0.000	0.000
θ_{22}	0.000	0.000	0.278	0.189	0.000	0.237	0.000
θ_{23}	0.000	1.000	0.151	0.207	0.445	0.000	0.000
θ_{24}	0.000	0.000	0.293	0.267	0.229	0.250	0.000
θ_{25}	0.000	0.000	0.000	0.221	0.127	0.368	0.000
θ_{26}	0.000	0.000	0.000	0.116	0.199	0.145	1.000

Note in table 3.14 the posterior probability for observing all states of the world following forecast Z_{21} is zero. The reason is that the forecast Z_{21} has not been observed in the past (table 3.12); hence, the posterior probability for θ_{2j} (j = 1, 2, ..., 6) is zero (table 3.14).

2. Computation of the no data and data Bayesian strategies

Thus far in chapter four the components of the Bayesian decision model have been defined in terms useful in assisting the cattle feeder in making feeding and marketing decisions while facing uncertainty, and in terms of the arithmetic procedures and equations used to quantify the components. This subsection illustrates the procedures, in equational form, for determining both Bayesian strategies using the notation previously illustrated.

Because of the nature of the cattle feeding-marketing decision problem involving multi-time period forecasts and production decision intervals, it was necessary to determine both the data and no data Bayesian strategy for each decision interval independently of the others. It was assumed that the variances of the price changes for two, four, six, and eight month intervals were different; therefore, a different prior $P(\theta)$ and likelihood $P(Z|\theta)$ were determined for each decision interval. Cattle price predictions two months in advance appeared more frequently than predictions of four or six months, and four month predictions appeared more frequently than longer periods. Determining the optimal action for each decision interval separately allowed the inclusion of all available cattle information for the specific period of interest without the same updated data being available for the other intervals.

Two types of Bayesian strategies were determined at each decision point, the no data and data strategy. The no data strategy is the simpler of the two to determine and employs the prior probability distribution of states of the world in selecting the "optimal" act. The data strategy employs the posterior distribution. As discussed previously, the posterior distribution of states of the world is determined by combining the prior and sample information. The same payoff matrix is used to determine both strategies.

The first step in selecting the no data Bayesian strategy is to compute the expected gain for each action. The following series of equations were used to compute expected gains at each decision node t in the planning horizon.

$$EG'(A_{aooo}) = \sum_j G_{aoooj}\, P(\theta_{ij}) \quad \text{when} \qquad 2.3.1'$$

$t = 1$ then $i = 2$ and $j = 1, \ldots, 6$

$$EG'(A_{aboo}) = \sum_j G_{abooj}\, P(\theta_{ij}) \quad \text{when} \qquad 2.3.2'$$

$t = 1$ then $i = 4$ and $j = 1, \ldots, 7$

$t = 2$ then $i = 2$ and $j = 1, \ldots, 6$

$$EG'(A_{abco}) = \sum_j G_{abcoj}\, P(\theta_{ij}) \quad \text{when} \qquad 2.3.3'$$

$t = 1$ then $i = 6$ and $j = 1, \ldots, 7$

$t = 2$ then $i = 4$ and $j = 1, \ldots, 7$

$t = 3$ then $i = 2$ and $j = 1, \ldots, 6$

$$EG'(A_{abcd}) = \sum_j G_{abcdj}\, P(\theta_{ij}) \quad \text{when} \qquad 2.3.4'$$

$t = 1$ then $i = 8$ and $j = 1, \ldots, 6$

$t = 2$ then $i = 6$ and $j = 1, \ldots, 7$

$t = 3$ then $i = 4$ and $j = 1, \ldots, 7$

$t = 4$ then $i = 2$ and $j = 1, \ldots, 6$

The no data Bayesian strategy is to select the action within each decision interval or production period that maximizes expected gains, as shown by the following equations.

$$EG'(A'_{aooo}) = \max_{aooo} EG'(A_{aooo}) \qquad 2.5.1'$$

$$EG'(A'_{aboo}) = \max_{aboo} EG'(A_{aboo}) \qquad 2.5.2'$$

$$EG'(A'_{abco}) = \max_{abco} EG'(A_{abco}) \qquad 2.5.3'$$

$$EG'(A'_{abcd}) = \max_{abcd} EG'(A_{abcd}) \qquad 2.5.4'$$

The superscript ' on A' is used to identify the action that maximizes expected gain in each decision interval.

At each decision node the Bayesian no data strategy selected is the strategy from amongst all the no data strategies that maximize expected gains for each of the remaining production periods as shown by equation 3.32.

$$EG'(A^*) = \max_{'} EG'(A') \qquad 3.32$$

If the selected Bayesian no data strategy A* is not one of the feeding activities in the next production period, then the prerequisite feeding action to A* is followed.

Using equation 2.3.1', $P(\theta_{2j})$ from table 3.11 and the gains in table 3.10, the expected income from A_{0000}, A_{1000}, A_{2000}, and A_{4000} is \$0.0, \$-9.09, \$-18.88, \$-10.39, and \$-2.97, respectively. By equation 2.5.1' the no data Bayesian strategy is A_{0000} since $EG'(A_{0000})$ is greater than $EG'(A_{aooo})$ where a = 1, 2, 3, or 4.

The data Bayesian strategies are based on the posterior distribution. The expected gain for each act is computed by a variation of equation 2.4 as shown below at each of the t decision nodes.

$$EG''(A_{aoook}) = \sum_j G_{aoooj} P(\theta_{ij}|Z_{ik}) \qquad 2.4.1'$$

when t = 1 then i = 2; j = 1, ..., 6; k = 1, ..., 7

$$EG''(A_{abook}) = \sum_j G_{abooj} P(\theta_{ij}|Z_{ik}) \qquad 2.4.2'$$

when t = 1 then i = 4; j = 1, ..., 7; k = 1, ..., 7

t = 2 then i = 2; j = 1, ..., 6; k = 1, ..., 7

$$EG''(A_{abcok}) = \sum_j G_{abcoj} P(\theta_{ij}|Z_{ik}) \qquad 2.4.3'$$

when t = 1 then i = 6; j = 1, ..., 7; k = 1, ..., 7

t = 2 then i = 4; j = 1, ..., 7; k = 1, ..., 7

t = 3 then i = 2; j = 1, ..., 6; k = 1, ..., 7

$$EG''(A_{abcdk}) = \sum_j G_{abcdj} P(\theta_{ij}|Z_{ik}) \qquad 2.4.4'$$

when $t = 1$ then $i = 8$; $j = 1, \ldots, 6$; $k = 1, \ldots, 7$

$t = 2$ then $i = 6$; $j = 1, \ldots, 7$; $k = 1, \ldots, 7$

$t = 3$ then $i = 4$; $j = 1, \ldots, 7$; $k = 1, \ldots, 7$

$t = 4$ then $i = 2$; $j = 1, \ldots, 6$; $k = 1, \ldots, 7$

The data Bayesian strategy is to select the action in each production period that maximizes expected income conditional on observing experiment results Z_{ik}. This is shown by the following series of equations.

$$EG''(A''_{aoook}) = \max_{aooo} EG''(A_{aoook}) \qquad 2.6.1'$$

$$EG''(A''_{abook}) = \max_{aboo} EG''(A_{abook}) \qquad 2.6.2'$$

$$EG''(A''_{abcok}) = \max_{abco} EG''(A_{abcok}) \qquad 2.6.3'$$

$$EG''(A''_{abcdk}) = \max_{abcd} EG''(A_{abcdk}) \qquad 2.6.4'$$

The superscript notation " identifies the action within each production period that maximizes expected gains for each Z_{ik} outcome.

After the Bayesian strategies have been selected for each remaining production period in the planning horizon, it is necessary to select one of the Bayesian strategies to follow. The Bayesian data strategy selected is the strategy with the highest expected gain as shown in equation 3.33.

$$EG''(A^{**}) = \max_{''} EG''(A'') \qquad 3.33$$

Similar to procedures described for the no data problem, if A** is not one of the feeding activities in the next production period, then the prerequisite feeding action to A** is followed.

The results of calculating data Bayesian strategies for activities in the first production period at the initial decision node are shown in table 3.15. The expected gain for each combination action-state of the world was calculated using the gains in table 3.10, the posterior distribution in table 3.14 and equation 2.4.1'. The second from bottom line in table 3.15 shows the expected gain from the Bayesian data strategy for each Z_{2k} forecast (equation 2.6.1'). The last line of table 3.15 shows the marginal probability of observing Z_{2k}.

Experiment outcome Z_{21} has not been observed in the past; as previously discussed, $P(\theta_{2j}|Z_{21})$ is equal to zero for all states of the world (table 3.14). The expected gains for all feeding activities conditional on observing sample result Z_{21} is zero. It seems reasonable to the author that if forecast Z_{21} is observed in the future the Bayesian strategy associated with observation Z_{22} would be followed. As discussed earlier, this is a conservative action since the forecast price increase of Z_{21} is greater than the increase for Z_{22}.

3. Value of market information

Difficulties are encountered when using normal Bayesian procedures as discussed in the previous chapter for determining the value of sample or market information in this dissertation as a result of multi-production periods or decision intervals in the planning horizon. Equations 2.7 through 2.12 were modified as shown below for the multi-decision interval model.

Table 3.15. [EG(A_{aooo}), P(θ_{2j}|Z_{2k})]; expected gain for all A_{aooo} given the Z_{2k}-th price forecast and the Bayesian strategy for each Z_{2k} forecast, December 1971

Action	EG"[A_{aooo}, P(θ_{2j}|Z_{2k})] Z_{2k}: Z_{21}[a]	Z_{22}	Z_{23}	Z_{24}	Z_{25}	Z_{26}	Z_{27}
				dollars			
A_{0000}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A_{1000}	0.00	-3.69	5.55	-9.91	-12.54	-12.22	-29.91
A_{2000}	0.00	-13.28	-3.68	-19.73	-22.45	-22.12	-40.49
A_{3000}	0.00	-4.58	5.36	-11.27	-14.10	-13.76	-32.79
A_{4000}	0.00	3.04	13.34	-3.89	-6.81	-6.45	-26.17
A''_{aoook}	--	A_{4000}	A_{4000}	A_{0000}	A_{0000}	A_{0000}	A_{0000}
EG"(A''_{aoook})	--	3.04	13.34	0.00	0.00	0.00	0.00
P(Z_{2k})	0.000	0.021	0.140	0.411	0.239	0.165	0.024

[a]Sample result Z_{21} has not been observed in the past; therefore, the Bayesian data strategy for experiment outcome Z_{21} is undefined.

The value of perfect information was determined at each of the t decision nodes for i months into the future by use of the following series of equations.

$$G(A^g_{aooo}|\theta_{ij}) = \max_{aooo} G(A_{aooo}|\theta_{ij}) \qquad 2.7.1'$$

when t = 1 then i = 2 and j = 1, ..., 6

$$G(A^g_{aboo}|\theta_{ij}) = \max_{aboo} G(A_{aboo}|\theta_{ij}) \qquad 2.7.2'$$

when t = 1 then i = 4 and j = 1, ..., 7

t = 2 then i = 2 and j = 1, ..., 6

$$G(A^{g}_{abco}|\theta_{ij}) = \max_{abco} G(A_{abco}|\theta_{ij}) \quad 2.7.3'$$

when t = 1 then i = 6 and j = 1, ..., 7

t = 2 then i = 4 and j = 1, ..., 7

t = 3 then i = 2 and j = 1, ..., 6

$$G(A^{g}_{abcd}|\theta_{ij}) = \max_{abcd} G(A_{abcd}|\theta_{ij}) \quad 2.7.4'$$

when t = 1 then i = 8 and j = 1, ..., 6

t = 2 then i = 6 and j = 1, ..., 7

t = 3 then i = 4 and j = 1, ..., 7

t = 4 then i = 2 and j = 1, ..., 6

Equation 2.8 was modified as follows to determine the conditional value of perfect information at each decision node t, i months into the future.

$$CVPI_{t+i}|\theta_{ij} = G(A^{g}_{aooo}|\theta_{ij}) - G(A'_{aooo}|\theta_{ij}) \quad 2.8.1'$$

when t = 1 then i = 2 and j = 1, ..., 6

$$CVPI_{t+i}|\theta_{ij} = G(A^{g}_{aboo}|\theta_{ij}) - G(A'_{aboo}|\theta_{ij}) \quad 2.8.2'$$

when t = 1 then i = 4 and j = 1, ..., 7

t = 2 then i = 2 and j = 1, ..., 6

$$CVPI_{t+i}|\theta_{ij} = G(A^{g}_{abco}|\theta_{ij}) - G(A'_{abco}|\theta_{ij}) \quad 2.8.3'$$

when t = 1 then i = 6 and j = 1, ..., 7

t = 2 then i = 4 and j = 1, ..., 7

t = 3 then i = 2 and j = 1, ..., 6

$$CVPI_{t+i}|\theta_{ij} = G(A^{g}_{abcd}|\theta_{ij}) - G(A'_{abcd}|\theta_{ij}) \qquad 2.8.4'$$

when t = 1 then i = 8 and j = 1, ..., 6

t = 2 then i = 6 and j = 1, ..., 7

t = 3 then i = 4 and j = 1, ..., 7

t = 4 then i = 2 and j = 1, ..., 6

The expected value of perfect market information (EVPI) was determined by weighting $CVPI|\theta_{ij}$ by the prior probability of observing θ_{ij} at each decision node t, i months into the future. Equation 2.9 was changed as follows for each of the four decision nodes.

Decision node 1:

$$EVPI_{t+i} = \sum_{j} P(\theta_{ij}) \; (CVPI_{t+i}|\theta_{ij}) \qquad 2.9.1'$$

when t = 1 then i = 2 and j = 1, ..., 6

Decision node 2:

$$EVPI_{t+i} = \sum_{j} P(\theta_{ij}) \; (CVPI_{t+i}|\theta_{ij}) \qquad 2.9.2'$$

when t = 1 then i = 4 and j = 1, ..., 7

t = 2 then i = 6 and j = 1, ..., 6

Decision node 3:

$$EVPI_{t+i} = \sum_{j} P(\theta_{ij}) \; (CVPI_{t+i}|\theta_{ij}) \qquad 2.9.3'$$

when t = 1 then i = 6 and j = 1, ..., 7

t = 2 then i = 4 and j = 1, ..., 7

t = 3 then i = 2 and j = 1, ..., 6

Decision node 4:

$$EVPI_{t+i} = \sum_j P(\theta_{ij})\ (CVPI_{t+i}|\theta_{ij}) \qquad 2.9.4'$$

when $t = 1$ then $i = 8$ and $j = 1, \ldots, 6$

$t = 2$ then $i = 6$ and $j = 1, \ldots, 7$

$t = 3$ then $i = 4$ and $j = 1, \ldots, 7$

$t = 4$ then $i = 2$ and $j = 1, \ldots, 6$

The conditional value of sample information (CVSI) is dependent on the particular sample results observed (Z_{ik}) and the no data Bayesian strategy A'. Equation 2.10 was modified as follows.

$$CVSI_{t+i}|Z_{ik} = EG''(A''_{aoook}) - EG''(A'_{aooo}) \qquad 2.10.1'$$

when $t = 1$ then $i = 2$ and $k = 1, \ldots, 7$

$$CVSI_{t+i}|Z_{ik} = EG''(A''_{abook}) - EG''(A'_{aboo}) \qquad 2.10.2'$$

when $t = 1$ then $i = 4$ and $k = 1, \ldots, 7$

$t = 2$ then $i = 2$ and $k = 1, \ldots, 7$

$$CVSI_{t+i}|Z_{ik} = EG''(A''_{abcok}) - EG''(A'_{abco}) \qquad 2.10.3'$$

when $t = 1$ then $i = 6$ and $k = 1, \ldots, 7$

$t = 2$ then $i = 4$ and $k = 1, \ldots, 7$

$t = 3$ then $i = 2$ and $k = 1, \ldots, 7$

$$CVSI_{t+i}|Z_{ik} = EG''(A''_{abcdk}) - EG''(A'_{abcd}) \qquad 2.10.4'$$

when $t = 1$ then $i = 8$ and $k = 1, \ldots, 7$

$t = 2$ then $i = 6$ and $k = 1, \ldots, 7$

$t = 3$ then $i = 4$ and $k = 1, \ldots, 7$

$t = 4$ then $i = 2$ and $k = 1, \ldots, 7$

As shown by equation 2.11, expected value of sample information can be computed prior to observing sample results Z_{ik} by weighting $(CVSI|Z_{ik})$ by $P(Z_{ik})$ and summing over Z. Equation 2.11 was modified to calculate EVSI for cattle forecasts i months into the future from each decision node.

Decision node 1:

$$EVSI_{t+i} = \sum_k P(Z_{ik})\ (CVSI_{t+i}|Z_{ik}) \qquad 2.11.1'$$

when t = 1 then i = 2 and k = 1, ..., 7

Decision node 2:

$$EVSI_{t+i} = \sum_k P(Z_{ik})\ (CVSI_{t+i}|Z_{ik}) \qquad 2.11.2'$$

when t = 1 then i = 4 and k = 1, ..., 7

t = 2 then i = 2 and k = 1, ..., 7

Decision node 3:

$$EVSI_{t+i} = \sum_k P(Z_{ik})\ (CVSI_{t+i}|Z_{ik}) \qquad 2.11.3'$$

when t = 1 then i = 6 and k = 1, ..., 7

t = 2 then i = 4 and k = 1, ..., 7

t = 3 then i = 2 and k = 1, ..., 7

Decision node 4:

$$EVSI_{t+i} = \sum_k P(Z_{ik})\ (CVSI_{t+i}|Z_{ik}) \qquad 2.11.4'$$

when t = 1 then i = 8 and k = 1, ..., 7

t = 2 then i = 6 and k = 1, ..., 7

t = 3 then i = 4 and k = 1, ..., 7

t = 4 then i = 2 and k = 1, ..., 7

The payoffs reported in this dissertation are on a per head basis; therefore, in determining ENGS it was necessary to allocate the cost of the sample (CS) to all cattle in the feedlot (N). Equation 2.12 was changed to:

$$ENGS_{t+i} = EVSI_{t+i} - \frac{CS}{N} \qquad 2.12'$$

when $t = 1; i = 2, 4, 6, 8$

$t = 2; i = 2, 4, 6$

$t = 3; i = 2, 4$

$t = 4; i = 2$

$ENGS_{t+i}$ is the expected net gain of sample information at the t-th decision node, i months into the future. ENGS is the expected value of the sample information less the cost of obtaining it. When using the economic decision model described in this dissertation, it is possible to calculate as many as four ENGS values for one Outlook letter. One value would be assigned to each cattle forecast for periods of two, four, six, and eight months into the future at the first decision node in each planning horizon.

EVPI, EVSI, and ENGS were computed for each optimal Bayesian strategy selected during 1972 and 1973. Since the calculated values of information varied between decision points, selected statistical parameters used to represent all the respective values were calculated. The parameters were the mean, range, and variance. In addition, a test of significance (t test) was computed to determine which mean values were significantly different from zero.

The CVPI and EVPI can be calculated for a two month cattle forecast at the initial decision node using the payoffs reported for each combination action-state of the world in table 3.10, the prior for θ_{2j} ($j = 1, \ldots, 6$) in table 3.11, and the no data strategy (A_{0000}) obtained using equations 2.3.1' and 2.5.1'. By use of equation 2.7.1', with perfect knowledge of when each state of the world would occur, a decision maker would select the maximum elements in each column of table 3.10 as shown below.

$$G(A^g_{4000}|\theta_{21}) = \$36.43$$

$$G(A^g_{4000}|\theta_{22}) = \$15.65$$

$$G(A^g_{4000}|\theta_{23}) = \$3.04$$

$$G(A^g_{0000}|\theta_{2j}) = \$0.00 \qquad \text{where } j = 4, 5, 6$$

The CVPI (equation 2.8.1') is:

$$EVPI_{1+2} = P(A^g_{aooo}|\theta_{2j}) - P(A'_{oooo}|\theta_{2j})$$

$$\$36.43 = \$36.43 - \$0.00 \text{ for } \theta_{21}$$

$$\$15.65 = \$15.65 - \$0.00 \text{ for } \theta_{22}$$

$$\$3.04 = \$3.04 - \$0.00 \text{ for } \theta_{23}$$

$$\$0.00 = \$0.00 - \$0.00 \text{ for } \theta_{2j} \qquad (j = 4, 5, 6)$$

The EVPI is determined by weighting CVPI for each state of the world by the prior probability of observing each θ_{2j} as shown in equation 2.9.1".

$$EVPI_{1+2} = \sum_{j=1}^{j=6} P(\theta_{2j}) \; (CVSI_{1+2}|\theta_{2j}) = \$4.57 \qquad 2.9.1''$$

The procedures used to calculate CVSI and EVSI can be illustrated using data shown in table 3.15, equations 2.10.1' and 2.11.1', and the Bayesian no data solution (A_{4000}) obtained in 2.5.1'.

$$CVSI_{1+2} = EG''(A''_{aoook}) - EG''(A'_{aooo}) \text{ for all } Z_{2k} \text{ in } Z \qquad 2.10.1''$$

$3.04 = $3.04 - $0.00 for k = 2

$13.34 = $13.34 - $0.00 for k = 3

$0.00 = $0.00 - $0.00 for k = 4, 5, 6, 7

EVSI using equation 2.11.1', $P(Z_{2k})$ is obtained from table 3.15.

$$EVSI_{1+2} = \sum_{k=2}^{k=7} P(Z_{2k}) \; (CVSI_{1+2}|Z_{2k}) = \$1.93 \qquad 2.11.1''$$

Assuming the 300 head feedlot was filled to 90 percent capacity, the ENGS is $1.92 per head from following the Bayesian data strategy at the first decision node, December 1971.

$$ENGS_{1+2} = EVSI_{1+2} - \frac{CS}{N} \qquad 2.12'$$

$$\$1.92 = \$1.93 - \frac{\$2.00}{270}$$

H. The "Naive" Model

The "naive" model, unlike the Bayesian strategies, was not an economic or statistical model but was a model that assumed no uncertainty in livestock prices, did not allow for variation in ration composition, or variation in length of time cattle were retained on feed as a function of feed ingredient costs and livestock price expectations. The naive model assumed that prices forecast in the Iowa Farm Outlook letter would prevail at market time. The cattle were fed a total ration that consisted of:

50.00 bushels shelled corn (89% DM)

1.83 tons of corn silage (40% DM)

30 pounds of soybean oil meal

69 pounds of urea supplement

Assumptions utilized in formulating the ration were that a pound of gain could be obtained by feeding 6.6 pounds dry matter of corn or 9.3 pounds dry matter of corn silage. It was assumed that a yearling steer could not consume more than one percent of body weight in corn per day nor more than 2.3 percent body weight in corn silage per day. The cattle were purchased at 750 pounds and marketed at a maximum weight of 1184 pounds. Rate of gain was 2.70 pounds per day, and animals were retained in the feedlot for a maximum of 180 days. Nonfeed variable costs and total revenue were calculated the same as nonfeed variable costs and revenue for the economic model. The ration costs per day used in calculating total feed costs are shown in table A.3.

I. Time Period for Testing the Model

Extreme variation in cattle feeding profits have occurred during the 1970's. In 1971, 1972, and during the first half of 1973, cattle feeding was extremely profitable. In the last half of 1973 and all of 1974 cattle feeders incurred heavy losses. The latter period was unique in the number of exogenous factors that occurred which had serious economic effects on the cattle feeding industry and the mid-west farmer-feeder. Government wage and price ceilings, consumers' beef boycott, the unprecedented reduction of total United States feed stocks, and the Middle East oil boycott were some of the factors that directly or indirectly affected the profitability of feeding cattle. Following is a brief summary of these "shocks" to the system.

In August 1971 a 90 day freeze (phase I) on almost all prices, wages, and rents was announced by the federal government. Unprocessed raw foods and agricultural commodities were exempt from the freeze. In November of the same year phase II of an economic program to control inflation was implemented. Prices were permitted to rise but only by an amount enough to compensate for increase in costs of production; profit margins were limited to the highest level that existed in a specified base period. As in phase I, unprocessed agricultural products were excluded from the regulations.

In January 1973, phase III was initiated. Mandatory controls on wages and prices were ended by President Nixon except for problem areas such as food, health, and construction industry. Phase I was replaced by a program which was self-administering and based on voluntary compliance. The administration did, however, retain authority to set mandatory rules when it appeared necessary.

Three months after phase III was started, the Cost of Living Council observed meat prices going up faster than costs to the meat packer, so meat packers and processors were placed under a mandatory program permitting dollar-for-dollar pass-through of increases and decreases in raw material costs.

On March 29, 1973, meat price ceilings were imposed on beef, pork, and lamb at wholesale and retail levels but not on farmers' live animal sales (47). The administration was seeking help from the "housewife rebellion" in combating the high retail price of beef. The following week a national beef boycott was initiated. The impact of the boycott was that meat sales were off up to 80 percent from the previous week in some stores (48). Retail stores decreased beef orders, thus forcing some packers to suspend operations; hence, live cattle prices fell.

In August 1973, phase IV lifted all the mandatory price ceilings except on beef. The following month, ceilings on beef prices were lifted.

Feed grain prices were pushed upward by the reduction in feed grain reserves and the poor weather that affected much of the mid-western United States during early 1973. A poor anchovy harvest in the Pacific during this same period created upward pressures on soybean prices. Anchovies and soybeans are both used in high protein animal feeds and are highly competitive in world markets.

The Middle East oil boycott in fall of 1973 caused increases in the cost of petroleum and petroleum derived products used in producing feed grains and livestock. Increase in petroleum prices increased costs of producing and marketing both feed grains and livestock.

The period used to test or implement the model developed in this dissertation was from January 1972 through December 1973. As discussed previously, this period includes both good and bad times in terms of profitability for the cattle feeder. It provides what the author feels are extremes for testing the economic model. The monthly average price for choice feeder steers varied from $35.82 to $56.39 per hundredweight, choice slaughter steers varied from $35.17 to $52.55 per hundredweight. The price of corn more than doubled during this period ranging from $1.04 to $2.31 per bushel, while soybean oilmeal and cottonseed oil meal varied from $5.60 to $20.80 and $5.80 to $14.00 per hundredweight, respectively.

IV. RESULTS

The following chapter reports the results of an economic decision model developed that incorporates livestock price uncertainty to assist the cattle feeder in making feeding and marketing decisions. Some decisions the cattle feeder makes are whether to feed or not feed cattle, what ration to feed, what rate of gain to achieve, how long to feed cattle, and at what weight range to market the cattle. The outcome of these decisions is generally uncertain at the time when the decisions are made. Even though cattle have been placed on feed, the decision maker is continuously gaining additional price information and re-evaluating earlier feeding decisions. The interpretation given the most recent feed and livestock market information will determine whether the original feeding and marketing intentions will be followed or whether they will be changed. The objective of the economic decision model developed in this dissertation is to maximize returns above variable costs for a yearling feedlot operation when making feeding and marketing decisions under price uncertainty. Uncertainty is incorporated into the decision model by use of probability theory.

Two Bayesian decision models were incorporated into the economic decision model; the no data model made use of a prior distribution of price changes, the data model combined sample or forecast information with the prior to select the feeding and marketing actions that maximize expected gain. Price predictions appearing in the Iowa Farm Outlook letter provided sample or additional information to the decision maker.

A third and somewhat naive model developed assumed no forecasting error in the price predictions appearing in the Outlook letter.

It was assumed that predicted prices would prevail with complete certainty. In the latter model, ration composition was not allowed to vary as the relative or absolute price of feed ingredients changed.

A. Bayesian "No Data" Strategies

The planning horizon was divided into four 60 day decision intervals or production periods. A decision was made prior to each production period whether to feed, continue feeding, or sell cattle currently on feed. A separate Bayesian model was employed to select the optimal feeding strategy for each production period remaining in the planning horizon. The action selected was the one that maximized expected discounted returns above variable costs.

The results of the no data solutions appear in table 4.1. At the first decision node, December 1971, solutions to four Bayesian models were obtained. The strategies were identified that maximized expected gain for production periods two, four, six, and eight months into the future.

For the first two month production period, five feeding actions (A_{0000}, A_{1000}, A_{2000}, A_{3000}, A_{4000}) were considered. The expected returns for activities A_{1000}, A_{2000}, A_{3000}, and A_{4000} were negative. The action that maximized expected return for the two month production period was A_{0000}.

Feeding activities considered for four months (two production periods) in the future were A_{1100}, A_{1200}, ..., A_{4400}. The no data solution is A_{4400} which had a discounted expected return of $31.35 per head. The feeding activities considered for six months into the future were activities A_{1110}, A_{1120}, ..., A_{4440}. Of the 26 activities

Table 4.1. Bayesian "no data" strategies, expected payoffs, and actions followed to maximize expected returns above variable costs for Iowa cattle feeders, 1972-1973

Date	Months in the future	Decision node	Optimal strategy	Expected gain	Subsequent feeding action	Accrued return for cattle on feed
	no.	no.	activity	$/head	activity	$/head
December 31, 1971	2	1	0000	0.00		--
	4	1	4400	31.35		
	6	1	4430	56.87		
	8	1	3333	60.09	3000	
February 29, 1972	2	2	3400	44.43		-6.86
	4	2	3440	74.44		
	6	2	3333	76.61	3300	
April 30, 1972	2	3	3340	51.92		22.06
	4	3	3333	58.02	3330	
June 30, 1972	2	4	3333	93.84	3333	81.19
July 31, 1972	2	1	0000	0.00		--
	4	1	4400	44.90		
	6	1	4430	73.56		
	8	1	3333	77.85	3000	
September 30, 1972	2	2	3400	5.30		2.80
	4	2	3440	35.44		
	6	2	3333	40.98	3300	
November 30, 1972	2	3	3340	13.75		-11.89
	4	3	3333	18.45	3330	

Table 4.1. Continued

Date	Months in the future	Decision node	Optimal strategy	Expected gain	Subsequent feeding action	Accrued return for cattle on feed
	no.	no.	activity	$/head	activity	$/head
January 31, 1973	2	4	3333	94.33	3333	80.98
February 28, 1973	2	1	0000	0.00		--
	4	1	4400	38.48		
	6	1	4430	70.72		
	8	1	3333	76.73	3000	
April 30, 1973	2	2	3400	41.71		-10.21
	4	2	3440	82.82		
	6	2	3333	88.26	3300	
June 30, 1973	2	3	3340	97.53		52.61
	4	3	3333	112.41	3330	
August 31, 1973	2	4	3333	171.48	3333	157.49
September 30, 1973	2	1	0000	0.00		--
	4	1	2200	19.27		
	6	1	4330	54.02		
	8	1	2233	71.73	2000	
November 30, 1973	2	2	2200	-6.11	2200	-24.02
	4	2	0000	0.00		
	6	2	0000	0.00		

considered that required three production periods, A_{4430} had the highest discounted expected return of \$56.87, and therefore was selected as the Bayesian no data strategy. Activities considered for eight months into the future were A_{1111}, A_{1112}, ..., A_{4322}. Note that the last activity in table 3.1 is A_{4322}. A_{3333} was the Bayesian no data strategy with the largest expected return, \$60.09 per head.

A_{3333} was selected from among the four Bayesian strategies at decision node one, since its expected return exceeded that of the other Bayesian no data strategies. The feeding action followed during the subsequent production period was A_{3000}. This action is the first of a combination of actions that leads to A_{3333} in the fourth production period.

After the first production period in February 1972, the decision maker re-evaluated the decision to feed ultimately for A_{3333}. At decision node two the cattle feeder needs only to consider activities feasible in the remaining production periods. All feasible activities have as a prerequisite A_{3000}. Activities feasible in the second production period are A_{3000}, A_{3200}, A_{3300}, and A_{3400}. Activities feasible in the production period ending four months in the future are A_{3210}, A_{3220}, ..., A_{3440}. Likewise, feasible activities in the last production period in the planning horizon are A_{3211}, A_{3212}, ..., A_{3333}.

At decision node two in the planning horizon three solutions to the Bayesian models were determined, one for each of the remaining production periods. The Bayesian no data strategies (table 4.1) for the three remaining production periods were A_{3400}, A_{3440}, and A_{3333}, respectively. The expected returns associated with the optimal actions were \$44.43, \$74.44, and \$76.61 per head, respectively. The action that maximizes expected return was A_{3333}.

The last column of table 4.1 shows the accrued income for cattle currently on feed at decision node two to be a minus \$6.86 per head. This is the net gain that would be realized if the cattle were sold. The expected return of \$76.61 associated with A_{3333} is greater than current realizable return; therefore, the decision was made to continue feeding cattle. A_{3300} was selected since it is the second in a series of actions leading to A_{3333}.

At the end of second production period, decision node three, realizable return was \$22.06 per head. Again, earlier decisions were re-evaluated using the economic decision model. The feasible activities considered at decision node three have as a prerequisite A_{3300}. In the third production period the feasible activities considered were A_{3320}, ..., A_{3340}. The feasible activities for the last production period were A_{3321}, ..., A_{3333}. The optimal no data actions in the third and fourth production periods were A_{3340} and A_{3333}. The expected returns for the respective activities were \$51.92 and \$58.02 per head.

Since the expected return of \$58.02 from continued feeding exceeds the realizable return of \$22.06, the decision was made to continue feeding and the action selected was A_{3330}. A net return of \$81.19 was realizable at the end of the third production period.

Again at decision node four the economic model was utilized to determine the expected gain from continued feeding. The two remaining feasible actions were A_{3332} and A_{3333}. The optimal action was A_{3333} and had an expected return of $93.84 per head.

Since the expected return from continued feeding exceeded realizable return of $81.19 associated with A_{3330s}, the cattle were retained on feed. The cattle were sold when the maximum allowable weight of 1200 pounds was reached (assumption 4). The maximum weight was attained after 21 days of feeding in the fourth production period. The realized income above variable costs was $94.75 per head (table 4.2).

The accumulated variable costs incurred and revenue realizable or received for each feeding activity followed during the two year sample period are shown in table 4.2. The accumulated costs for the first group of cattle fed were $335.29 per head, revenue received was $430.04 per head, and an income above variable expenses was $94.75 per head.

The last column of table 4.2 shows the realizable return for each feeding activity at the end of the respective production period. The realizable returns in table 4.2 are equivalent to the figures shown in the last column of table 4.1.

A second planning horizon started on July 31, 1972. Procedures used to determine optimal feeding and marketing actions were identical to those described for the first planning horizon. Activity A_{3333} was the optimal feeding action at each of the four decision nodes. The realized return at the end of the second planning horizon in February 1973 was $124.62 per head (table 4.2). The expected return at the start of the fourth production period was $94.33.

Table 4.2. Returns above variable expenses from following Bayesian "no data" strategies for Iowa cattle feeders, 1972-1973

Date	Feeding action	Purchase price	Ration cost	Nonfeed cost	Accumulated total cost	Revenue at end of feeding period: Realizable	Revenue at end of feeding period: Received	Income above variable expenses at end of feeding period: Realizable	Income above variable expenses at end of feeding period: Received
	activity				$/head				
Dec. 31, 1971	3000	255.54	16.14	6.00	277.68	270.82		-6.86	
Feb. 29, 1972	3300		17.58	6.00	301.26	323.32		22.06	
April 30, 1972	3330		18.96	6.00	326.22	407.41		81.19	
June 30, 1972	3333		6.97	2.10	335.29		430.04		94.75
July 31, 1972	3000	282.45	16.44	6.00	304.89	307.69		2.80	
Sept. 30, 1972	3300		18.00	6.00	328.89	317.00		-11.89	
Nov. 30, 1972	3330		21.18	6.00	356.07	437.05		80.98	
Jan. 31, 1973	3333		7.85	2.10	366.02		490.64		124.62
Feb. 28, 1973	3000	337.42	18.60	6.00	362.02	351.81		-10.21	
April 30, 1973	3300		20.34	6.00	388.36	440.97		52.61	
June 30, 1973	3330		21.96	6.00	416.32	573.81		157.49	
August 31, 1973	3333		7.92	2.10	426.34		505.84		79.50
Sept. 30, 1973	2000	357.52	14.22	6.00	377.74	353.72		-24.02	
Nov. 30, 1973	2200[a]		12.72	3.00	393.46		322.70		-70.76

[a]Ration and nonfeed variable costs are computed for a 30 day period rather than 60 days to coincide with the end of the calendar year and test period.

The third planning horizon started in February 1973. Again A_{3333} was the optimal feeding action at each of the four decision nodes. The expected returns were \$76.73, \$88.26, \$112.41, and \$171.48, respectively. The realized income above variable costs for the third group of cattle was \$79.50 per head when sold in September 1973.

The last planning horizon in the two year period commenced in September 1973. The expected return of \$71.73 for A_{2233} was greater than the expected return for the other three Bayesian solutions at the first decision node (table 4.1). Realizable income at the end of the first production period was \$-24.02 per head.

The results of the three Bayesian no data decision models at decision node two (November 1973) were unique. The expected returns from continued feeding were negative for all production periods remaining in the planning horizon. The Bayesian no data strategy was to sell the cattle currently on feed (A_{0000}). The losses from continued feeding for the three production periods were \$6.11, \$35.38, and \$23.78, respectively. The expected loss from action A_{2200} was less than the realizable loss of \$24.02 per head if the cattle were sold at the beginning of the second production period. The minimum loss was associated with feeding one additional production period. The decision to retain cattle on feed was made since the expected loss of \$6.11 for A_{2200} was less than the realizable loss of \$24.02 if the cattle on feed were sold. Expected losses were minimized by continued feeding. The actual losses incurred by feeding until the end of 1973 were \$70.76 per head (table 4.2).

The pattern of optimal feeding strategies for the four decision nodes during the first three planning horizons were identical. At the first decision node activities A_{0000}, A_{4400}, A_{4430}, and A_{3333} were the optimal strategies for the four remaining production periods in the planning horizon. Action A_{3333} consistently had the greatest expected gain. The expected return of A_{3333} never exceeded A_{4430} by more than \$6.01 per head. The expected return of A_{4430} ranged between 1.63 and 1.83 times the expected return of A_{4400}.

If a decision criteria of maximizing average net returns per day rather than maximizing returns from each pen of cattle had been used initially, a different feeding action would have been selected at decision node one. The average expected daily net returns of A_{4430} were 32, 42, and 39 cents per day, respectively, at the first decision node in planning horizons one, two, and three. At the same decision nodes the average expected daily net returns for A_{3333} were 30, 39, and 38 cents, respectively.

At decision node two in the first three planning horizons, A_{3400}, A_{3440}, and A_{3333} consistently maximized expected returns for the second, third, and fourth production periods, respectively. The expected return of A_{3333} always exceeded returns of other feeding actions.

The pattern continued at decision node three in planning horizons one, two, and three. A_{3340} and A_{3333} maximized expected returns for the last two production periods. Similar to previous decision nodes, the expected return of A_{3333} exceeded that of other Bayesian strategies. Only two actions were feasible at decision node four. The expected return of A_{3333} was greater than the expected return of A_{3332}.

The last planning horizon was unique in that the optimal actions were to feed cattle at a slower daily rate of gain than in the previous feeding periods. During the first twelve production periods cattle were fed a ration that would provide adequate energy to gain 2.5 pounds per day. A gain of 2.0 pounds per day was achieved during the last two production periods.

B. Bayesian "Data" Strategies

The second and somewhat more sophisticated economic model developed to assist the cattle feeder in making production and marketing decisions requires the use of a Bayesian data decision model. The Bayesian data model combines data or sample information and the prior probability to form a posterior probability distribution for the states of the world. The posterior distribution was used to determine the action with the highest expected payoff.

The results of 37 data Bayesian decision models used to determine feeding and marketing strategies during the sample period are shown in table 4.3. The most recent price forecast appearing in the Iowa Outlook letter is represented by the Z_{ik} value shown in the fourth column. Each Z_{ik} value represents a specific forecast for a change in the level of choice slaughter steers. The period of time associated with the forecast is shown in the third column. A forecast for each state of the world is represented by a unique Z_{ik} value. Only the Bayesian data strategy associated with the Z_{ik}-th forecast appearing in the Outlook letter is reported. A Z_{ik} value when k is less than four indicates a forecast increase in choice slaughter cattle prices; a

Table 4.3. Bayesian "data" strategies, expected payoffs, and actions followed to maximize returns above variable costs for Iowa cattle feeders, 1972-1973

Date	Decision node	Months in the future	Outlook forecast	Bayesian strategy	Expected gain	Subsequent feeding action	Accrued return for cattle on feed
	no.	no.	Z_k	activity	$/head	activity	$/head
December 31, 1971	1	2	6	0000	0.00		--
	1	4	6	4400	30.97		
	1	6	6	4430	46.89		
	1	8	5	3333	49.50	3000	
February 29, 1972	2	2	6	3400	44.19		-6.86
	2	4	7	3440	33.96		
	2	6	6	3333	65.92	3300	
April 30, 1972	3	2	6	3340	50.59		22.06
	3	4	6	3333	59.15	3330	
June 30, 1972	4	2	6	3333	94.80	3333	81.19
July 31, 1972	1	2	6	0000	0.00		--
	1	4	7	4400	18.55		
	1	6	7[a]	0000	0.00		
	1	8	7[a,b]	3333	70.59	3000	
September 30, 1972	2	2	4	3400	3.22		2.80
	2	4	4	3440	31.56		
	2	6	4	3333	48.44	3300	

[a]No price forecast was available within the past 60 days; assumed prices predicted for four months would prevail.

[b]Z_{87} was undefined; Bayesian strategy for Z_{86} was selected.

Table 4.3. Continued

Date	Decision node	Months in the future	Outlook forecast	Bayesian strategy	Expected gain	Subsequent feeding action	Accrued return for cattle on feed
	no.	no.	Z_k	activity	$/head	activity	$/head
November 30, 1972	3	2	3	3340	35.42		-11.89
	3	4	3	3333	40.69	3330	
January 30, 1973	4	2	6	3333	93.78	3333	80.98
February 28, 1973	1	2	6[c]	0000	0.00		0.00
	1	4	6[c]	4400	40.38		
	1	6	7[c]	0000	0.00		
	1	8	7[c,b]	3333	68.66	3000	
April 30, 1973	2	2	6	3400	40.98		-10.21
	2	4	6	3440	83.75		
	2	6	7	3333	86.65	3300	
June 30, 1973	3	2	6[c]	3340	100.08		52.61
	3	4	6[c]	3333	113.51	3330	
August 30, 1973	4	2	4[c]	3333	169.17	3333	157.49
September 30, 1973	1	2	2	0000	0.00		0.00
	1	4	1[d]	2200	17.37		
	1	6	1	4330	50.13		
	1	8	3	2233	64.16	2000	

[c]Price forecast was made within the past 60 days.

[d]Z_{41} was undefined; Bayesian strategy for Z_{42} was selected.

Table 4.3. Continued

Date	Decision node	Months in the future	Outlook forecast	Bayesian strategy	Expected gain	Subsequent feeding action	Accrued return for cattle on feed
	no.	no.	Z_k	activity	$/head	activity	$/head
November 30, 1973	2	2	4	2200[e]	-8.04	2200	-24.02
	2	4	1[d]	0000	0.00		
	2	6	6	0000	0.00		

[e]Bayesian strategy was A_{0000}; however, expected gain from A_{2200} would minimize losses.

Z_{ik} value when k is greater than four indicates a forecast decrease in choice slaughter cattle prices.

The basic format of table 4.3 is similar to that used in table 4.1 for the no data problem. The accrued returns for cattle currently on feed, if any, are shown in the last column of table 4.3.

The procedures described in the previous section for the no data problem when deciding whether to feed, continue feeding cattle, or market cattle currently on feed were used in the data problem. If expected return exceeded the current realizable return at each decision node, cattle were purchased for feeding, or continued on feed; otherwise, the decision to sell or not feed was implemented.

At decision node one, July 1972, price forecasts were not available for six or eight month periods (Z_{6k} and Z_{8k}). Since forecasts for these periods did not appear within the past 60 days, it was assumed prices predicted for four months (Z_{47}) would prevail for the following four months, or until the end of the planning horizon ($Z_{47} = Z_{67} = Z_{87}$). Using the four month price forecast, the Bayesian strategy for the six month period was not to feed or action A_{0000}. The forecast Z_{87} was undefined for the eight month period; therefore, the Bayesian strategy associated with Z_{86} was followed. The Bayesian strategy for Z_{86} was A_{3333}.

Further difficulties were not encountered with price forecasts until February 1973. January forecasts were utilized for all periods. Forecast Z_{87} was undefined for the eight month period; the strategy for Z_{86} was substituted. Similar substitutions of forecasts were made in June and August of 1973. In September and November of 1973, forecast

Z_{41} was undefined for the forecasts four months into the future. The Bayesian strategy for Z_{42} was selected for each period.

Similar to the no data solutions at the last decision node in the sample period, the Bayesian data strategy was A_{0000} for all periods remaining in the planning horizon. Even though the expected return for A_{2200} was negative, cattle were continued on feed because the expected loss of $8.04 per head was less than the current realizable loss of $24.02 per head.

The Bayesian data strategies were the same as the strategies selected when using the Bayesian no data model except in July 1972 and February 1973. The no data strategy for the six month period from decision node one, July 1972, was A_{4430}; the data strategy associated with a price decrease Z_{67} was not to feed cattle or A_{0000}. An identical situation developed in February 1973; the no data Bayesian strategy was A_{4430}; the data strategy for Z_{67} was A_{0000} for the six month period from decision node one.

C. Value of Market Information

The value of perfect market information, as defined in chapter two and illustrated in chapter three, is computed by observing the true state of the world and then finding the difference between returns resulting from following the no data strategy and the gains resulting from the action which would have been selected if the true state of the world had been known in advance with certainty. The expected value of perfect market information (EVPI) is obtained by weighting the value of perfect information for each state of the world by the prior probability of observing each state of the world.

The value of sample information was defined as the difference between expected gains from following the data strategy after observing the Z_{ik}-th experiment or sample results and the expected gains from following the no data strategy. The expected value of sample information (EVSI) is determined prior to observing an outcome of experiment Z. EVSI is determined by weighting the value of sample information for all outcomes of Z by the marginal probability of observing each Z_{ik}-th sample result.

Both EVPI and EVSI are functions of the prior and posterior distributions of states of the world, respectively, and of elements in the payoff matrix. EVPI and EVSI were determined for each of the 37 data and no data problems, since the data utilized in computing both the prior and posterior distributions were continuously updated. In addition, payoffs reported in the payoff matrix were continuously changing due to different feasible actions at the various decision nodes and different base prices for cattle and ration costs. The estimates of selected population parameters, tests of hypotheses for EVPI and EVSI, and other descriptive characteristics are shown in tables 4.4 and 4.5. The parent population for EVPI and EVSI are choice slaughter cattle price forecasts appearing in the Iowa Farm Outlook letter for periods of two, four, six, and eight months.

The range of values observed for EVPI are shown in table 4.4 for all forecast periods and for the optimal Bayesian strategies selected at each of the 14 decision nodes. EVPI varied between zero and \$5.99 per animal.

Table 4.4. Descriptive and statistical measures for expected value of perfect market information, yearling feeding program, Iowa, 1972-1973

Measure	Units	Bayesian strategy followed	Forecast period months 2	4	6	8
Range of values						
High	$/head	4.62	5.99	3.52	3.68	0.00
Low	$/head	0.00	0.00	0.00	0.00	0.00
Observations						
Total	no.	14	14	11	8	4
EVPI > 0	no.	4	8	8	6	0
Mean	$/head	0.67	2.11	0.99	0.73	0
Standard deviation		1.480	2.361	1.147	1.239	0
t value for Ho:μ=0		.453	.894	.863	.590	--[a]
Probability of a larger value of t		.17	.12	.13	.15	--[a]

[a]Undefined.

A zero value for EVPI occurred whenever one action dominated other feasible actions over all states of the world. The prior probability distribution of states of the world did not influence the selection of a feeding action when one action dominated. The decision maker needed only to select the dominant action to maximize expected returns. The dominant action is the Bayesian strategy and also the action the decision maker would select if he had perfect information as to the true state of the world. The EVPI at decision node one for all feeding and marketing activities eight months in the future and some feeding and marketing activities at other decision nodes was zero as a result of one action dominating.

Table 4.5. Statistical and descriptive measures for expected value of sample information for choice slaughter cattle and expected net gain from sampling, Iowa Farm Outlook letter, 1972-1973

Measure	Units	Bayesian strategy followed	Forecast period months: 2	4	6	8
Range of values						
High	$/head	1.69	1.94	1.69	0.83	0.00
Low	$/head	0.00	0.00	0.00	0.00	0.00
Observations						
Total	no.	14	14	11	8	4
EVSI > 0	no.	3	7	3	4	0
Mean	$/head	0.26	0.53	0.26	0.18	0.00
Standard deviation		0.548	0.747	0.567	0.305	0.00
t value for Ho:μ=0		0.474	0.710	0.459	0.590	--[a]
Probability of a larger value of t		0.17	0.14	0.17	0.16	--[a]
Range of ENGS						
High	$/head	1.68	1.93	1.68	0.82	-0.01
Low	$/head	-0.01	-0.01	-0.01	-0.01	-0.01

[a]Undefined.

The mean, standard deviation, and calculated t values of EVPI for each forecast period are presented in table 4.4. The probability of observing a larger t value is shown in the last line of the table. Probability levels are for a one tail test.

Similar to EVPI, statistical and descriptive measures for EVSI are shown in table 4.5. The range of values for EVSI appear for all periods and for the optimal Bayesian data strategies selected. EVSI for all eight month forecasts were zero. The range of values for EVSI varied from zero to $1.94 per head. The highest value for each production period decreased as length of price forecast increased.

The mean values for EVSI, standard deviation, and t test to test the null hypothesis that the mean is equal to zero are presented for each forecast period. EVSI indicates the expected increase in returns over a long run period that the decision maker could expect to receive from following data strategies.

The range of values for expected net gain of sampling are shown for each forecast period. ENGS varied from a minus one cent to a positive $1.93 per head per observation.

D. "Naive" Decision Model

Cattle were assumed to gain an average of 2.7 pounds per day and be retained on feed for a maximum of 180 days. The planning horizon for the naive model is divided into three 60 day production periods. Cattle were placed or continued on feed whenever the anticipated returns exceeded the present realizable returns. The anticipated returns calculated at all decision nodes are shown in table 4.6.

Using the naive model, anticipated returns for all production periods were negative at the first decision node in the test period. In January 1972, feeder cattle were purchased, since the anticipated discounted returns associated with feeding cattle for six months were $12.39 per head. At the end of the first production period, the realizable returns above variable expenses were a minus $7.21 per head. The anticipated discounted returns from continued feeding for four months were $17.48. The decision was made to continue feeding, since the anticipated returns exceeded the realizable returns if the cattle were sold at decision node two.

Table 4.6. Anticipated revenue, returns, and variable expenses associated with the "naive" cattle feeding and marketing decision model, Iowa, 1972-1973

Date	Months in the future	Decision node	Variable expenses for each production period: Purchase	Ration	Nonfeed	Accumulated total	Anticipated revenue	Anticipated discounted returns	Accrued returns for cattle on feed
	no.	no.	$/head						
Dec. 31, 1971	2	1	255.54	24.21	6.00	285.75	238.14	-47.14	--
	4	1		25.80	6.00	317.55	292.09	-24.96	
	6	1		24.95	6.00	348.50	347.33	-1.14	
Jan. 31, 1972	2	1	251.60	24.40	6.00	282.00	249.36	-32.32	--
	4	1		25.82	6.00	313.82	311.76	-2.02	
	6	1		24.74	6.00	344.56	357.32	12.39	
March 31, 1972	2	2	251.60	50.43	12.00	314.03	317.88	3.81	-7.21
	4[a]	2		24.95	6.00	344.98	362.81	17.48	
May 31, 1972	2	3	251.60	76.41	18.00	346.01	381.09	34.73	30.64
July 31, 1972	2	1	282.45	25.27	6.00	313.72	279.85	-33.53	--
	4	1		27.07	6.00	346.79	318.58	-27.66	
	6[b]	1		25.98	6.00	378.77	369.05	-9.44	
Aug. 31, 1972	2[a]	1	280.44	25.21	6.00	311.65	287.93	-23.49	--
	4[a]	1		26.91	6.00	344.56	317.84	-26.20	
	6[b]	1		25.78	6.00	376.34	369.57	-6.57	

[a]Anticipated revenue is based on a price forecast made within the previous two months.

[b]No six month price forecast was available during the previous two months. It was assumed that prices predicted for four months would prevail in six months.

Table 4.6. Continued

Date	Months in the future	Decision node	Variable expenses for each production period: Purchase	Ration	Nonfeed	Accumulated total	Anticipated revenue	Anticipated discounted returns	Accrued returns for cattle on feed
	no.	no.	$/head						
Sept. 30, 1972	2	1	285.45	25.91	6.00	317.36	299.40	-17.78	--
	4	1		28.17	6.00	351.53	338.04	-13.23	
	6	1		27.22	6.00	384.75	393.77	8.76	
Nov. 30, 1972	2	2	285.45	54.30	12.00	351.75	345.48	-6.21	-7.23
	4	2		26.96	6.00	384.71	399.08	14.09	
Jan. 31, 1973	2	3	285.45	84.57	18.00	388.02	432.24	43.70	37.58
March 31, 1973	2^a	1	353.48	30.01	6.00	389.49	310.49	-78.07	--
	4^a	1		31.81	6.00	427.30	370.98	-55.01	
	6^a	1		29.85	6.00	463.15	412.57	-48.83	
April 30, 1973	2	1	342.39	30.19	6.00	378.58	342.14	-36.01	--
	4	1		32.27	6.00	416.85	410.03	-6.66	
	6	1		30.47	6.00	453.32	436.55	-16.19	
May 31, 1973	2^a	1	365.08	32.93	6.00	404.01	343.33	-59.97	--
	4^a	1		36.33	6.00	446.34	411.00	-34.52	
	6^a	1		34.60	6.00	486.94	437.41	-47.82	
June 30, 1973	2^a	1	355.35	37.85	6.00	399.20	353.41	-45.25	--
	4^a	1		44.32	6.00	449.52	411.29	-37.34	
	6^a	1		43.28	6.00	498.80	438.87	-57.86	
July 31, 1973	2	1	365.49	37.20	6.00	408.69	401.38	-7.22	--
	4	1		44.24	6.00	458.93	460.41	1.45	
	6^b	1		43.90	6.00	508.83	535.08	25.34	

Table 4.6. Continued

Date	Months in the future	Decision node	Variable expenses for each production period: Purchase	Ration	Nonfeed	Accumulated total	Anticipated revenue	Anticipated discounted returns	Accrued returns for cattle on feed
	no.	no.	$/head						
Sept. 30, 1973	2	2	365.49	81.84	12.00	459.33	454.94	-4.34	-23.04
	4	2		44.73	6.00	510.06	574.11	62.55	
Nov. 30, 1973	2	3	365.49	130.31	18.00	513.80	451.96	-61.11	-80.41

The decision was made to continue feeding at decision node three. The realizable returns were \$30.64 and the anticipated returns were \$34.73 per head. The first group of cattle was sold in July 1972; the realized return was \$78.34 per head (table 4.7).

The costs incurred from feeding, revenue, and realizable and realized returns are shown for each feeding period in table 4.7. The realizable returns in table 4.7 are equivalent to the accrued returns appearing in the last column of table 4.6.

Replacement feeder cattle were not purchased until the following September, at which time the anticipated returns were \$8.76 per head for feeding for six months. The cattle were sold at the end of the planning horizon. The realized returns were \$123.28 per head (table 4.7).

The third and last group of cattle was purchased in July 1973. The anticipated return at the first decision node was \$25.34 per head for feeding for six months (table 4.7). At decision node two, anticipated returns of \$62.55 associated with continued feeding exceeded the realizable returns of a minus \$23.04 per head.

In November 1973, at decision node three, the anticipated returns from continued feeding were a minus \$61.11 per head. The decision was made to continue feeding since the anticipated loss was less than the realizable loss (\$80.41) if the cattle were sold. It appeared that losses would be minimized by continued feeding. The actual loss at the end of the test period was \$83.37 per head.

Table 4.7. Variable expenses, revenue, and returns for feeding cattle when using the "naive" cattle feeding and marketing decision model, Iowa, 1972-1973

Date	Feeding decision[a]	Purchase	Ration	Nonfeed	Accumulated total	Revenue at end of period		Income above variable expense	
						Realizable	Received	Realizable	Received
				$/head					
Dec. 31, 1971	-								
Jan. 31, 1972	+	251.60	24.40	6.00	282.00	274.79		-7.21	
March 31, 1972	+		26.03	6.00	314.03	344.67		30.64	
May 31, 1972	+		25.98	6.00	346.01		424.35		78.34
July 31, 1972	-								
Aug. 31, 1972	-								
Sept. 30, 1972	+	285.45	25.91	6.00	317.36	310.13		-7.23	
Nov. 30, 1972	+		28.39	6.00	351.75	389.33		37.58	
Jan. 31, 1973	+		30.27	6.00	388.02		511.30		123.28
March 31, 1973	-								
April 30, 1973	-								
May 31, 1973	-								
June 30, 1973	-								
July 31, 1973	+	365.49	37.20	6.00	408.69	385.65		-23.04	
Sept. 30, 1973	+		44.64	6.00	459.33	378.92		-80.41	
Nov. 30, 1973	+		24.24[b]	3.00[b]	486.57	403.20			-83.37

[a]Positive or negative sign indicates cattle were fed or not fed, respectively, during the subsequent feeding period.

[b]Ration and nonfeed variable costs are computed for a 30 day period rather than 60 days to coincide with the end of the calendar year and test period.

E. Comparison of the Decision Models

The primary objective of the decision models developed was to maximize returns above variable costs to the Iowa cattle feeder. Perhaps the best way to evaluate the different decision models is to compare realized returns between models for the two year test period.

The net return per animal was higher for the economic decision model which utilized probability theory and allowed for variation in rate of gain and ration composition than for the naive model. The net returns received when following the data strategies were identical to the net gains received when following the no data strategies. The accumulated net gains received for the two year test period from following the economic decision model strategies were $228.11 per head; the accumulated gains from following the naive model were $118.25 per head. Assuming the 300 head feedlot was filled to 90 percent capacity for each group of cattle fed, total gains attributed to the economic decision model and the naive decision model for the two year test period were $61,589.70 and $31,927.50, respectively. The net gain from following the strategies resulting from the economic model was $29,622.20 greater than gains from the naive model.

Livestock were fed to achieve two rates of gain (2.0 and 2.5 pounds per day) following economic decision model strategies and only one rate (2.7 pounds per day) following the naive model. The two pound rate of gain was achieved during the latter part of the test period. During this time feed prices were higher than experienced earlier and cattle prices were decreasing.

V. SUMMARY AND CONCLUSIONS

A cattle feeder is confronted with making many decisions for which the outcome is not known with certainty at the time the decision is made. Prior to placing cattle on feed the cattle feeder has some idea or expectation concerning future cattle and feed prices and the performance ability of cattle intended to be purchased. It is not until the cattle are marketed that the results of the earlier production and marketing decisions can be realized.

A. Summary of Problem and Procedures

The primary objective of this dissertation was to develop an economic model that would assist the cattle feeder in making production and marketing decisions under uncertainty. Perhaps the greatest area of uncertainty facing the cattle feeder is that of future livestock prices. The direction of price changes influences the profitability of cattle feeding. Between 1965 and 1971 choice slaughter cattle prices in Iowa varied as much as \$5.37 per hundredweight over a two month period; between 1971 and 1973 prices varied up to \$12.04 per hundredweight between two month periods. During the latter period net returns ranged from \$161.00 loss to \$130.00 profit per head. By anticipating price changes and the direction of price changes, a cattle feeder can re-evaluate and alter earlier production and marketing plans to maximize expected profits or minimize expected losses if cattle are currently being fed.

The economic decision model developed in the dissertation incorporated several production and marketing factors to assist the cattle feeder in maximizing expected returns above variable costs for a yearling feedlot operation. Initially, 88 feeding activities were considered which allowed the cattle feeder to feed for various rates of daily gain and to vary the length of time cattle are retained on feed. Rates of gain considered varied in one-half pound increments from 1.5 to 3.0 pounds per day. Cattle were allowed to remain on feed for a maximum of 240 days or until a weight of 1200 pounds was attained.

Basic to the economic decision model was the development of a least cost ration formulation submodel. A linear program was used to determine the least cost ration for each feeding activity. Seven feed ingredients were considered in formulating each ration. Basic constraints were placed on protein, roughage, urea, and dry matter intake. Net energy requirements were determined as a function of daily gain and metabolic weight of the animal being fed. Feed ingredient prices were based on prices prevailing in Iowa at the time the rations were formulated. Corn silage was priced on the basis of opportunity income of corn grain at the time of harvest.

Returns above variable costs were calculated for all feasible feeding activities at four decision nodes during the 240 day planning horizon. The decision nodes were 60 days apart. In addition to ration costs, other costs considered were purchase price of a 750 pound steer, nonfeed variable costs such as veterinary and medicine, death loss, waste handling, etc. Nonfeed variable costs were obtained from secondary sources assuming a 300 head capacity feedlot.

Average cattle prices for respective weight groups and quality grade of cattle prevailing in Iowa at each decision node were utilized to calculate total returns. As cattle increase in weight, it was assumed that quality grade increases. Regression analysis was used to estimate the proportion of cattle that graded choice or higher for each weight interval considered. Beef carcass data reports obtained from the Animal Science Department, Iowa State University, provided data utilized in the regression analysis.

The optimal feeding actions were determined at each decision node for all remaining production periods in the planning horizon by use of Bayesian decision models. Both data and no data strategies were determined. In each case the strategy selected was the strategy from amongst the Bayesian strategies for the remaining production periods that maximized expected income.

States of the world considered in the Bayesian model were changes in the average price of choice grade slaughter steers in interior Iowa for periods of two, four, six, and eight months. The prior probability distribution for states of the world was determined by calculating the empirical frequency with which the respective states had occurred since June 1965.

Basic to the data Bayesian decision model is sample or forecast information. The forecast is utilized to predict the state of the world. Rather than develop a forecast model specifically for this dissertation, the decision was made to utilize forecast information available to any Iowa cattle feeder. The Iowa Farm Outlook letter, published by the Iowa Cooperative Extension Service two times a month, was used to obtain forecast information. The conditional probability of observing each possible forecast given the true state of the world was determined using Outlook forecasts since 1965. The conditional probability or likelihood was combined with the prior probability by use of Bayes' theorem to compute a posterior probability for observing each state of the world given a specific price forecast.

The prior probability of states of the world was utilized to select the feeding action that maximized expected returns for the no data Bayesian decision model. The data Bayesian decision model utilized the posterior probabilities. Associated with results of both Bayesian decision models were the least cost ration, optimal rate of gain, and expected payoff of the selected action.

The expected value of perfect market information, expected value of sample information appearing in the Outlook letter, and expected net gain of sample information were determined at each decision node for all production periods remaining in the planning horizon.

The derived income from following the economic decision model strategies was compared to the income received from following a naive feeding and marketing decision model. The latter model assumes no forecast error exists in the Outlook letter and allows no variation in rate of gain or ration composition as feed ingredient costs or livestock price expectations change. The decision criterion used in both the economic and the naive decision models was to feed or continue feeding so long as net returns could be increased or losses minimized.

B. Summary of Results

The accumulated net return for the 1972-1973 test period derived from following strategies associated with the economic decision model was greater than the return from the naive decision model. The feeding strategies associated with the data and no data Bayesian decision models were identical; therefore, the realized net returns were the same.

The accumulated net returns for the economic decision model were $228.11 per head of feedlot capacity. Three groups of cattle were fed and marketed during the test period. A fourth group had been on feed for three months at the end of the test period. Two daily rates of gain were fed to maximize expected returns. A 2.5 pound daily rate of gain was attained for the first three groups, and a 2.0 pound daily rate for the last group of cattle fed.

In the naive decision model strategies, three groups of cattle were placed on feed during the two year sample period. The first group was fed for 180 days, and realized gains were $78.34 per head. The second group was fed 180 days, and realized gains were $123.28 per head. The third group of cattle was still on feed at the end of the test period; accrued losses were $83.37 per head. The accumulated net gain was $118.25 per head of feedlot capacity for the 24 month period.

The observed expected value of perfect market information had high values of $5.99, $3.52, and $3.68 per head per observation for production periods of two, four, and six months. Similarly, the observed expected value of sample information contained in the Iowa

Farm Outlook letter had high values of \$1.94, \$1.69, and \$.83 per head per observation for cattle forecasts two, four, and six months. The expected value of perfect market information and sample information was zero for all eight month forecast periods and some two, four, and six month periods. A zero value resulted when one of the feasible actions dominated all other actions considered in the particular Bayesian decision problem.

C. Conclusions on Procedures, Model, and Results

At the conclusion of any research project, it is always rewarding to discuss the strengths of the model used and/or positive results observed. It is, however, somewhat frustrating when one must point out the weaknesses of the model. Following is a summary of both strengths and weaknesses observed. The author will attempt to avoid use of the supposition "if" when discussing weaknesses of the model.

The economic decision model developed in the dissertation included several basic variables that the cattle feeder needs to consider when making decisions concerning feeding and marketing of livestock. Variables such as rate of gain, length of time to feed, feed ingredient costs, least cost rations, nonfeed variable costs, feedlot performances, and prices for both feeder and slaughter livestock were included in the model. Price uncertainty associated with marketing livestock was incorporated into the economic model via a Bayesian decision model. The incorporation of uncertainty is perhaps the greatest asset of the economic model.

The economic model is not so sophisticated that it could not be adapted and utilized by some of the more progressive cattle feeders.

There are several aspects of the model that could be improved before future use. One might want to reduce the time between decision nodes. The cattle feeder is continuously updating and re-evaluating feeding and marketing decisions rather than every 60 days. The decision process is a continuous process, not discrete as treated in the economic and naive models.

Feed ingredient and livestock prices were based on prevailing monthly average prices during the sample period. The variances associated with the prices utilized were not included in the model. It is conceivable that greater price fluctuation occurred within some months than between months.

Problems were encountered in using cattle price forecasts appearing in the Outlook letter. First, forecasts were not always available at the time needed nor for the required production periods. Secondly, the Outlook letter contained forecasts for choice slaughter weight steers. It was necessary to estimate prices for other quality grade and weight groups using current known price relationships. These relationships can and do change over time.

The prices used to represent various states of the world should have covered a wider range of price changes. A wider range of states of the world would have prevented one feeding action from dominating all other activities. It must be recognized, however, that never before has the cattle industry experienced such drastic price changes as occurred in 1973. Problems would have been encountered if a wider range of price changes were considered; both the assigned prior and conditional probability of observing these changes would have been zero.

Additional problems were encountered when using Bayesian procedures to determine expected value of sample information contained in the Outlook letter and expected net gain of the sample information. First, typically with Bayesian decision models, only one sample result is observed from an experiment when making a preposterior decision. If the decision is made to sample, the outcome is observed and the Bayesian strategy followed. In the economic model developed in this dissertation, it was possible to observe up to four experimental outcomes at each decision node. At decision node one, two, three, and four the number of sample observations observed were four, three, two, and one, respectively. Prior to observing all results, no one forecast could be allocated unambiguously or stated as the forecast that would be used in determining the Bayesian strategy. Therefore, the expected value of sample information was determined for each forecast period remaining in the planning horizon and a range of values for expected value of sample information were presented.

Secondly, the Outlook letter is obtained by paying an annual subscription fee, not a fee for each forecast observed. The subscription fee entitles the subscriber to receive all forecast information published during the year. At the time the subscription fee is paid, the subscriber is uncertain as to the number of forecasts that will be needed or be observed during the year. In normal Bayesian procedures the cost of obtaining the forecast is subtracted from the expected value of sample information for each forecast observed to determine expected net gain of the sample. In this dissertation the annual subscription fee was charged at each decision node for all forecasts remaining in the planning horizon. Similar to EVSI, a range of values was presented for ENGS.

Concluding remarks on results of the model are that the optimal rate of gain and length of time to retain cattle in the feedlot do change as feed ingredient prices, price relationships between weight groups and quality grades, and slaughter cattle price expectations change if profits are to be maximized. Utilizing the economic model yielded greater returns over the 1972-1973 test period than did the naive decision model. For the second objective, a specific figure for the value of cattle forecast information contained in the Iowa Farm Outlook letter was not arrived at.

D. Need for Further Research

It would be of interest to expand the economic decision model developed in this dissertation to consider different decision criteria. One such criterion would be to maximize expected long run profits rather than maximize profits from each pen of cattle fed. The model could be expanded to consider other type feeding programs, allow animals of different weight grou and sex to be fed. Consideration should be given to optimal replacement patterns.

The economic decision model allowed for variation in daily rate of gain, length of time cattle were retained on feed, variation in ration composition as a function of feed ingredient prices, and incorporated livestock price uncertainty to maximize expected returns. Further research needs to be completed to determine the effect of each of these variables on returns.

A survey of cattle feeders needs to be conducted to determine the decision criteria used. If the observed criterion is different than the assumed criteria used in this model, then adapt this model to increase its usefulness.

Recent trends in the midwest have been for a greater proportion of the cattle to be sold on a quality and yield grade basis. Data needs to be collected concerning price differentials paid for various yield grades. Rather than use quality grade and weight to determine price alone, include yield grade as a variable.

Additional research needs are to develop a procedure to determine the economic value of forecast information contained in a publication such as the Iowa Farm Outlook letter. One such procedure might be to modify the naive model by allowing ration composition to change as feed ingredient prices vary and assume no forecast error exists in the Outlook letter. The net returns realized from this model could then be compared to the net returns from a similar model developed in this dissertation.

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VIII. APPENDIX

Table A.1. Feed ingredient prices used in computing ration costs, Iowa, 1971-1973 (dollars per unit)

Ingredient	Units	Month					
		Dec.	Jan.	Feb.	March	April	May
		1971			1972		
Alfalfa	ton	27.00	27.50	28.00	28.00	27.50	26.00
Corn	bu.	1.15	1.14	1.14	1.15	1.18	1.20
Corn silage	ton	8.82	8.82	8.82	8.82	8.82	8.82
Cottonseed meal	cwt.	5.80	5.80	5.90	5.80	6.00	6.00
Grain sorghum	cwt.	2.15	2.14	2.14	2.14	2.16	2.18
Soybean meal	cwt.	5.50	5.60	5.70	5.90	6.20	6.20
Urea	ton	78.00	78.00	78.00	78.00	88.20	85.80
					1973		
Alfalfa	ton		30.50	31.50	31.00	32.00	30.00
Corn	bu.		1.40	1.36	1.38	1.41	1.61
Corn silage	ton		9.94	9.94	9.94	9.94	9.94
Cottonseed meal	cwt.		9.60	10.50	11.00	9.90	12.00
Grain sorghum	cwt.		3.05	2.81	2.80	2.69	2.79
Soybean meal	cwt.		10.70	11.70	12.70	11.80	16.20
Urea	ton		89.70	95.40	108.00	110.40	110.40

June	July	Aug.	Sept.	Oct.	Nov.	Dec.
25.50	26.00	26.50	27.00	27.50	27.00	28.50
1.19	1.20	1.19	1.26	1.20	1.24	1.45
8.82	8.82	8.82	8.82	9.94	9.94	9.94
6.00	6.00	6.00	6.20	6.40	6.90	8.60
2.17	2.18	2.21	2.36	2.35	2.47	3.05
6.30	6.50	6.70	6.80	6.90	7.50	9.70
85.80	84.00	84.00	84.00	84.00	84.00	86.40
29.50	28.00	31.00	31.50	32.00	35.00	37.00
2.03	2.06	2.75	2.10	2.16	2.24	2.41
9.94	9.94	9.94	9.94	16.24	16.24	16.24
14.00	12.50	13.50	13.00	12.50	11.50	12.50
3.24	3.36	4.68	3.92	3.93	3.83	3.95
20.80	15.60	18.50	13.00	12.00	10.60	11.70
110.40	110.40	110.40	110.40	110.40	110.40	110.40

June	July	Aug.	Sept.	Oct.	Nov.	Dec.
25.50	26.00	26.50	27.00	27.50	27.00	28.50
1.19	1.20	1.19	1.26	1.20	1.24	1.45
8.82	8.82	8.82	8.82	9.94	9.94	9.94
6.00	6.00	6.00	6.20	6.40	6.90	8.60
2.17	2.18	2.21	2.36	2.35	2.47	3.05
6.30	6.50	6.70	6.80	6.90	7.50	9.70
85.80	84.00	84.00	84.00	84.00	84.00	86.40
29.50	28.00	31.00	31.50	32.00	35.00	37.00
2.03	2.06	2.75	2.10	2.16	2.24	2.41
9.94	9.94	9.94	9.94	16.24	16.24	16.24
14.00	12.50	13.50	13.00	12.50	11.50	12.50
3.24	3.36	4.68	3.92	3.93	3.83	3.95
20.80	15.60	18.50	13.00	12.00	10.60	11.70
110.40	110.40	110.40	110.40	110.40	110.40	110.40

Table A.2. Cost per day of least cost rations for 88 cattle feeding activities considered in the economic decision model, Iowa, 1971-1973 (dollars)

Activity	1971	1972						1973						
	Dec.	Feb.	Apr.	June	July	Sept.	Nov.	Jan.	Feb.	Apr.	June	Aug.	Sept.	Nov.
1000	.176	.176	.177	.177	.176	.176	.198	.199	.199	.200	.200	.200	.200	.322
2000	.208	.208	.209	.209	.208	.208	.234	.235	.235	.236	.236	.236	.237	.380
3000	.269	.269	.273	.274	.274	.279	.300	.313	.313	.310	.316	.366	.424	.505
4000	.348	.346	.355	.357	.358	.371	.381	.413	.406	.417	.542	.688	.556	.664
1100	.192	.192	.193	.192	.192	.192	.216	.216	.217	.218	.218	.218	.218	.351
1200	.226	.226	.227	.227	.227	.227	.254	.255	.256	.257	.257	.257	.257	.413
2100	.197	.197	.198	.198	.198	.198	.222	.223	.223	.224	.224	.224	.224	.360
2200	.232	.232	.233	.233	.233	.233	.261	.262	.262	.264	.264	.264	.264	.424
2300	.289	.228	.291	.292	.292	.295	.322	.331	.330	.334	.368	.408	.372	.535
3200	.238	.238	.239	.239	.239	.239	.268	.269	.269	.271	.271	.271	.271	.435
3300	.293	.293	.296	.296	.297	.300	.328	.336	.335	.339	.370	.405	.373	.544
3400	.382	.380	.388	.390	.392	.403	.420	.450	.443	.454	.570	.705	.583	.724
4300	.297	.297	.300	.300	.300	.303	.333	.340	.339	.343	.369	.398	.372	.550
4400	.388	.387	.395	.397	.398	.409	.427	.457	.450	.461	.575	.708	.588	.735
1110	.211	.211	.212	.212	.211	.211	.237	.238	.238	.240	.240	.240	.240	.383
1120	.244	.244	.245	.245	.244	.244	.274	.275	.276	.277	.277	.277	.277	.446
1210	.218	.218	.219	.219	.218	.218	.245	.246	.246	.248	.248	.248	.248	.394
1220	.249	.250	.251	.251	.250	.250	.281	.282	.282	.284	.284	.284	.284	.457
1230	.301	.301	.304	.304	.304	.306	.338	.344	.343	.346	.368	.392	.370	.556
2110	.218	.218	.219	.219	.218	.218	.245	.246	.246	.248	.248	.248	.248	.394
2120	.249	.250	.251	.251	.250	.250	.281	.282	.282	.284	.284	.284	.284	.457
2210	.225	.225	.226	.225	.225	.225	.253	.254	.254	.256	.256	.256	.256	.405
2220	.256	.256	.257	.257	.256	.256	.288	.288	.289	.291	.291	.291	.291	.467
2230	.306	.306	.308	.308	.308	.310	.343	.348	.348	.351	.368	.388	.370	.563
2320	.261	.261	.262	.262	.262	.262	.294	.295	.295	.297	.297	.297	.297	.478
2330	.309	.310	.312	.312	.312	.313	.348	.352	.352	.355	.367	.381	.368	.570
2340	.406	.405	.412	.414	.415	.426	.448	.476	.470	.480	.586	.709	.598	.766
3210	.231	.231	.233	.232	.232	.232	.260	.261	.262	.263	.263	.263	.263	.416
3220	.261	.261	.262	.262	.262	.262	.294	.295	.295	.297	.297	.297	.297	.478
3230	.309	.310	.312	.312	.312	.313	.348	.352	.352	.355	.367	.381	.368	.570

Table A.2. Continued

Activity	1971 Dec.	1972 Feb.	1972 Apr.	1972 June	1972 July	1972 Sept.	1972 Nov.	1973 Jan.	1973 Feb.	1973 Apr.	1973 June	1973 Aug.	1973 Sept.	1973 Nov.
3320	.267	.267	.268	.268	.268	.268	.300	.301	.302	.304	.304	.304	.304	.488
3330	.314	.314	.316	.315	.315	.316	.353	.356	.356	.358	.366	.375	.367	.576
3340	.412	.411	.418	.420	.421	.431	.455	.482	.476	.486	.590	.710	.601	.776
3430	.317	.317	.319	.319	.318	.319	.357	.358	.359	.361	.364	.366	.364	.581
3440	.417	.416	.424	.425	.426	.436	.461	.488	.482	.492	.594	.712	.605	.786
4320	.273	.273	.234	.274	.273	.273	.307	.308	.308	.310	.310	.310	.310	.498
4330	.317	.317	.319	.319	.318	.319	.357	.358	.359	.361	.364	.366	.364	.581
4340	.417	.416	.424	.425	.426	.436	.461	.488	.482	.492	.594	.712	.605	.786
4430	.322	.322	.323	.323	.322	.322	.362	.363	.364	.366	.366	.366	.366	.588
4440	.420	.419	.426	.428	.429	.439	.464	.491	.485	.495	.595	.711	.606	.790
1111	.231	.231	.233	.232	.232	.232	.260	.261	.262	.263	.263	.263	.263	.416
1112	.261	.261	.262	.262	.262	.262	.294	.295	.295	.297	.297	.297	.297	.478
1121	.238	.238	.239	.239	.239	.239	.268	.269	.269	.271	.271	.271	.271	.427
1122	.267	.267	.268	.268	.268	.268	.300	.301	.302	.304	.304	.304	.304	.488
1123	.314	.314	.316	.316	.315	.316	.353	.356	.356	.358	.366	.375	.367	.576
1211	.238	.238	.239	.239	.239	.239	.268	.269	.269	.271	.271	.271	.271	.427
1212	.267	.267	.268	.268	.268	.268	.300	.301	.302	.304	.304	.304	.304	.488
1221	.245	.245	.246	.246	.246	.246	.276	.277	.277	.279	.279	.279	.279	.438
1222	.273	.272	.274	.274	.273	.273	.307	.308	.308	.310	.310	.310	.310	.498
1223	.317	.317	.319	.319	.318	.319	.357	.358	.359	.361	.364	.366	.364	.581
1232	.278	.278	.279	.279	.279	.279	.313	.314	.315	.316	.316	.316	.316	.509
1233	.322	.322	.323	.323	.322	.322	.362	.363	.364	.366	.366	.366	.366	.588
1234	.420	.419	.426	.428	.429	.439	.464	.491	.485	.495	.595	.711	.606	.790
2111	.238	.238	.239	.239	.239	.239	.268	.269	.269	.271	.271	.271	.271	.427
2112	.267	.267	.268	.268	.268	.268	.300	.301	.302	.304	.304	.304	.304	.488
2121	.245	.245	.246	.246	.246	.246	.276	.277	.277	.279	.279	.279	.279	.438
2122	.273	.273	.274	.274	.273	.273	.307	.308	.308	.310	.310	.310	.310	.498
2123	.317	.317	.319	.319	.318	.319	.357	.358	.359	.361	.364	.366	.364	.581
2211	.245	.245	.246	.246	.246	.246	.276	.277	.277	.279	.279	.279	.279	.438
2212	.273	.273	.274	.274	.273	.273	.307	.308	.308	.310	.310	.310	.310	.498
2221	.252	.252	.253	.253	.253	.253	.284	.284	.285	.287	.287	.287	.287	.450

Table A.2. Continued

Activity	1971	1972						1973						
	Dec.	Feb.	Apr.	June	July	Sept.	Nov.	Jan.	Feb.	Apr.	June	Aug.	Sept.	Nov.
2222	.278	.278	.279	.279	.279	.279	.313	.314	.315	.316	.316	.316	.316	.509
2223	.322	.322	.323	.323	.322	.322	.362	.363	.364	.366	.366	.366	.366	.588
2232	.293	.283	.284	.284	.284	.284	.318	.319	.320	.322	.322	.322	.322	.517
2233	.325	.325	.326	.326	.326	.326	.366	.367	.368	.370	.370	.370	.370	.594
2234	.423	.421	.429	.430	.432	.441	.468	.494	.488	.498	.596	.710	.607	.795
2321	.259	.259	.260	.260	.260	.260	.292	.292	.293	.294	.294	.294	.295	.461
2322	.283	.283	.284	.284	.284	.284	.318	.319	.320	.322	.322	.322	.322	.517
2323	.325	.325	.326	.326	.326	.326	.366	.367	.368	.370	.370	.370	.370	.594
2332	.286	.286	.287	.287	.286	.286	.322	.322	.323	.325	.325	.325	.325	.522
2333	.328	.328	.330	.329	.329	.329	.369	.370	.371	.373	.373	.373	.373	.600
2334	.426	.424	.432	.433	.434	.444	.471	.496	.490	.500	.597	.709	.608	.800
2343	.331	.331	.333	.332	.332	.332	.373	.374	.375	.377	.377	.377	.377	.606
3211	.252	.252	.253	.253	.253	.253	.284	.284	.285	.287	.287	.287	.287	.450
3212	.278	.278	.279	.279	.279	.279	.313	.314	.315	.316	.316	.316	.316	.509
3221	.259	.259	.260	.260	.260	.260	.292	.292	.293	.294	.294	.294	.295	.461
3222	.283	.283	.284	.284	.284	.284	.318	.319	.320	.322	.322	.322	.322	.517
3223	.325	.325	.326	.326	.326	.326	.366	.367	.368	.370	.370	.370	.370	.594
3232	.286	.286	.287	.287	.286	.286	.322	.322	.323	.325	.325	.325	.325	.522
3233	.328	.328	.330	.329	.329	.329	.369	.370	.371	.373	.373	.373	.373	.600
3234	.426	.424	.432	.433	.434	.444	.471	.496	.490	.500	.597	.709	.608	.800
3321	.265	.265	.266	.266	.265	.265	.298	.299	.299	.301	.301	.301	.301	.470
3322	.286	.286	.287	.287	.286	.286	.322	.322	.323	.325	.325	.325	.325	.522
3323	.328	.328	.330	.329	.329	.329	.369	.370	.371	.373	.373	.373	.373	.600
3332	.289	.289	.290	.290	.290	.290	.325	.326	.326	.328	.328	.328	.329	.528
3333	.331	.331	.333	.332	.332	.332	.373	.374	.375	.377	.377	.377	.377	.606
4321	.268	.268	.270	.269	.269	.269	.302	.303	.303	.305	.305	.305	.305	.476
4322	.289	.289	.290	.290	.290	.290	.325	.326	.326	.328	.328	.328	.329	.528

Table A.3. Ration costs per day per production period of ration fed using the "naive" decision model, Iowa, 1971-1973 (dollars)

Month	Year	Production period 1	2	3
December	1971	.403	.430	.416
January	1972	.407	.430	.412
March	1972	.409	.434	.416
May	1972	.421	.451	.433
July	1972	.421	.451	.433
August	1972	.420	.449	.430
September	1972	.432	.469	.454
November	1972	.453	.473	.449
January	1973	.491	.528	.504
March	1973	.500	.530	.498
April	1973	.503	.538	.508
May	1973	.549	.606	.577
June	1973	.631	.739	.721
July	1973	.620	.737	.732
September	1973	.619	.744	.745
November	1973	.761	.827	.808